Sl. No.

CE1.1-R4 : DIGITAL SIGNAL PROCESSING

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Total Time : 3 Hours **Total Marks : 100**

1. (a) Define the energy of a signal, and compute the energy of signal $x(t) = \exp(-5\ t)\ u(t)$ where $u(t)$ is unit step function.

(b) Define the power of a signal, and compute the power of signal $x(t) = A\ \cos(\omega t + \theta)$.

(c) Compute the impulse response $h[n]$of a Linear Time Invariant (LTI) system which is characterised by the difference equation $y[n] = x[n] - x[\mathrm{n} - 1]$, where $y[n]$ is output and $x[n]$ is input.

(d) Determine if system defined by the input output relation $y[n] = 3x^2[n] - 2x[n-3]$ is linear or not.

(e) Determine if system defined by the input output relation $y[n] = \Sigma_{k=-\infty}^{n} x[k]$ is time-invariant or not.

(f) Determine if systems defined by the input output relations (*i*) $y(t) = x^2(t)$, (*ii*) $y[n] = x[n] + x[n+2]$, (*iii*) $y[n] = 1/x[n]$ are stable or not.

(g) Determine the output $y[n]$ of LTI system for input $x[n]$ if impulse response is $h[n] = 0.5\ \delta[n] - 0.5\ \delta[n-1]$. **(7x4)**

2. (a) Explain the properties of the Region of Convergence (ROC) of the z-transform.

(b) There are two sequences $x_1[n]$ and $x_2[n]$, show that the z-transform of the convolution of these two sequences is $X_1(z)X_2(z)$.

(c) Find the poles and zeros of the function $H(z) = -z(z+0.1)/(z^2 - 2.05\ z + 1)$. Then determine the inverse z-transform of function H(z). **(5+5+8)**

3. (a) Obtain the Discrete-Time Fourier Transform (DTFT) of the signal $x[n] = a^{|n|}$, $|a| < 1$.

(b) What is the Goertzel algorithm ? How its working is different from the FFT algorithm ?

(c) Obtain an output of a LTI system whose impulse response is $h[n] = \alpha^n u[n]$ and input is $x[n] = \beta^n u[n]$ where $|\alpha| < 1$, $|\beta| < 1$ and $u[n]$ unit step sequence. **(5+5+8)**

4. (a) Write expressions for Discrete Fourier Series (DFS) and inverse DFS of discrete-time periodic signals. Using the inverse DFS expression, derive expression for the DFS.

(b) List out the various key differences in between the TMS 320C40 and TMS 320C50 coprocessors use in signal processing.

(c) Show that the periodic convolution of two sequences corresponds to multiplication in DFS domain. **(8+4+6)**

5. (a) Find the Discrete Fourier Transform (DFT) coefficients of a sequence $x[n]=1$, $n=0, 1, 2$ and $x[n]=0$ otherwise.

(b) Explain with necessary expressions Fast Fourier Transform (FFT) algorithm decimation-in-time.

(c) Find the DFT of a sequence $x(n)=\{1, 1, 0, 0\}$ and find the IDFT of $Y(K)=\{1, 0, 1, 0\}$.

(4+7+7)

6. (a) Design a digital filter equivalent of a 2^{nd} order Butterworth low-pass filter with a cut-off frequency fc = 100 Hz and a sampling frequency fs = 1000 samples/sec. Derive the finite difference equation of the filter.

(b) Determine if the difference equation $y[n]=x[n]+2x[n-1]+3x[n-2]$ correspond to IIR or FIR systems. A system is assumed causal.

(c) Draw the block diagram and signal flow graph representations of a LTI system whose input $x[n]$ and output $y[n]$ satisfy the following difference equation :

$y[n]=-a_1 y[n-1]-a_2 y[n-2]+b_0 x[n]$ **(6+5+7)**

7. (a) In a multi-rate signal processing, output $y[n]$ of an up-sampler for input $x[n]$ is given by $y[n] = x\left[\frac{n}{2}\right]$, when n is even and $y[n]=0$ if n is odd. Obtain expressions for the z-transform and DTFT of output sequence $y[n]$.

(b) In a multi-rate signal processing, output $y[n]$ of a down-sampler for input $x[n]$ is given by $y[n]=x[2n]$. Obtain expressions for the z-transform and DTFT of output sequence $y[n]$.

(c) Compute the autocorrelation of the sequence $x[n]=a^n u[n]$, where $u[n]$ is unit step sequence.

(5+6+7)

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