No. of Printed Pages : **3**

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# C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

**NOTE :**
1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same questions should be answered together and in the same sequence.**

**Total Time : 3 Hours** **Total Marks : 100**

1. (a) State and explain the basic axioms of probability theory.
   (b) What is the probability of getting a sum of 22 or more when four dice are thrown ?
   (c) Use the two-phase method to solve
   Minimize $Z = x_1 + x_2$
   Subject to,
   $2x_1 + x_2 \geqslant 4,$
   $x_1 + 7x_2 \geqslant 7$
   $x_1, x_2 \geqslant 0.$
   (d) Let X and Y be jointly continuous random variables with joint PDF given as :
   $$f_{X,Y}(x, y) = \begin{cases} cx + 1 & x, y \geqslant 0,\ x + y < 1 \\ 0 & \text{otherwise} \end{cases}$$
   Find the value of c.
   (e) Suppose the calls in a telephone system arrive randomly at an exchange at the rate of 140 per hour. If there are a very large number of lines available to handle the calls which last an average of 3 minutes, what is the average number of lines in use ?
   (f) Toss three coins. Let X denote the number of heads on the first two and Y denote the number of heads on last two.
   (i) Find the joint distribution of X and Y.
   (ii) Find E[Y|X=1]
   (g) Find the value of the Fourier transform for the sigmoid function. **(7x4)**

2. (a) Consider the New Delhi International Airport. Assume that it has one runway which is used for arrivals only. Airplanes have been found to arrive at a rate of 10 per hour. The time (in minutes) taken for an airplane to land is assumed to follow exponential distribution with mean 3 minutes. Assume that arrivals follow a Poisson process. Without loss of generality, assume that the system modelled as a M/M/1 queueing system
   (i) What is the steady state probability that there is no waiting time to land ?
   (ii) What is the expected number of airplanes waiting to land ?
   (iii) Find the expected waiting time to land.
   (b) Using the Laplace transform find the solution for the following equations :
   (i) $\frac{\partial^4}{\partial t^4} y(t) = 6\delta(t-1)$ having initial condition $y(0) = 0$, $Dy(0) = 0$
   (ii) $y(t) = t + \int_0^t -y(\tau)\sin(-t+\tau)\,d\tau$ with initial condition $y(0) = a$, $Dy(0) = b$ **(9+9)**

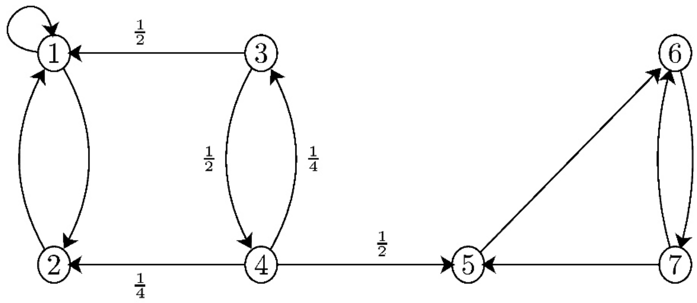
**3.** (a) Use the Kuhn-Tucker conditions to solve the non linear programming problem

Maximize $Z = 2x_1^2 - 7x_2^2 + 12x_1x_2$

Subject to,

$2x_1 + 5x_2 \leq 98.$

(b) Consider the Markov chain as given below. There are two recurrent classes, R1 = {1, 2}, and R2 = {5, 6, 7}. Assuming X0 = 3, find the probability that the chain gets absorbed in R1.




(c) Write down the stochastic condition of the following :

(i) A birth-and-death process is recurrent

(ii) A birth-and-death process is ergodic

(iii) A birth-and-death process is null-recurrent **(9+6+3)**

**4.** (a) Solve the following initial-value problem

$y' = 3e^x + x^2 - 4,\ y(0) = 5.$

(b) Suppose the given function is

$$f(t) = \begin{cases} 0 & \text{if } -\pi < t \leq 0, \\ \pi & \text{if } 0 < t \leq \pi. \end{cases}$$

Extend $f$(t)periodically and write it as a Fourier series. **(9+9)**

**5.** (a) Use branch and bound method to solve the following integer linear programming problem

Maximize $Z = 3x_1 + 4x_2$

Subject to,

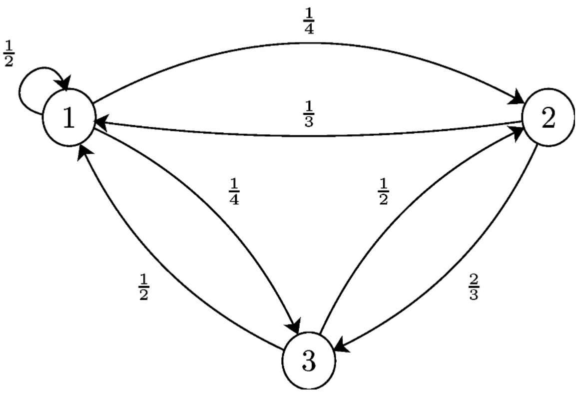
$7x_1 + 16x_2 \leq 52,$

$3x_1 - 2x_2 \leq 9$

$x_1, x_2 \geqslant 0.$

$x_1, x_2$ all are integers

(b) Consider the Markov chain shown in Figure.




Find :

(i) Is this chain irreducible ?

(ii) Is this chain aperiodic ?

(iii) Find the stationary distribution for this chain. **(12+6)**

**6.** (a) Find the Fourier Transform of the following function :

$$y(t) = \frac{d}{d(t)}\, te^{-3t}\, u(t) * e^{-zt}\, u(t)$$

(b) Using the Laplace Transform, evaluate

$$\int_0^{\infty} \frac{e^{-at}\sin^2 t}{t}\, dt$$ **(10+8)**

**7.** (a) Suppose that the time (in minutes) that a phone call lasts is a random variable with density function given by

$$f(t) = \begin{cases} \frac{1}{5}\, e^{-t/5}, & t > 0 \\ 0, & \text{otherwise} \end{cases}$$

Determine the probability that the phone call

(i) Takes longer than 5 minutes

(ii) Takes between 5 and 6 minutes

(iii) Takes less than 3 minutes

(iv) Takes less than 6 minutes given that it took at least 3 minutes.

(b) In a study, physicians were asked what the odds of breast cancer would be in a woman who was initially thought to have a 1% risk of cancer but who ended up with a positive mammogram result (a mammogram accurately classifies about 80% of cancerous tumors and 90% of benign tumors.) 95 out of a hundred physicians estimated the probability of cancer to be about 75%. Do you agree with the given statistics ? Prove using suitable probability measure ?

(c) Let X be a random variable such that

$P(x=1) = p = 1 - P(x = -1)$

Find a constant $c \neq 1$, such that $E[c^X] = 1$ **(6+6+6)**

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