

C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES**NOTE :**

1. Answer question no. 1 and any FOUR questions from 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Total Time : 3 Hours**Total Marks : 100**

1. (a) Find the rank of the matrix $\begin{bmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{bmatrix}$.

(b) Evaluate $\lim_{x \rightarrow \pi/4} \frac{\ln(\tan x)}{\sin x - \cos x}$.

(c) Verify whether the function $f(x) = (x^4 - 8x)$ satisfies Mean Value Theorem (MVT) in the interval $[0, 2]$.

(d) Determine the type of Conic for the following equation :
 $-4x^2 + 3y^2 - 8x + 24y + 32 = 0$
 Also, find the center of this Conic.

(e) Discuss the convergences of the infinite series $\sum_{n=1}^{\infty} \frac{2 \tan^{-1} n}{1+n^2}$.

(f) If $A \cap B = \phi$, then show that $P(A) \leq P(\bar{B})$.

(g) Are $Y = 5 + 2.5X$ and $Y = -3 + 2X$ be the estimated regression equations of Y and X and X on Y respectively. Explain your answer with suitable reason.

(7x4)

2. (a) Solve the equation $\begin{bmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{bmatrix} = 0$.

(b) Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

(c) Using Gauss elimination method, solve the system of simultaneous linear equations :
 $x - y + 2z = 7$
 $4x - 3y + z = 8$
 $-2x + y - 3z = -12$

(6+6+6)

3. (a) A company manufactures cylindrical metal drums with open tops with a volume of 2π cubic meters. What should be the dimensions of the drum in order to use the least amount of metal in their production ?
- (b) Find the area between the curve $y^2(2a-x) = x^3$ and its asymptote(s).
- (c) Find $\frac{dy}{dx}$ for the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. (8+6+4)

4. (a) Convert the polar equation $r = \frac{6}{2-3 \sin\theta}$ into rectangular coordinates.
- (b) Find the parametric equation of the circle $x^2 + y^2 - 2x + 4y - 20 = 0$.
- (c) Reduce the quadratic equation $2x^2 + 3y^2 + 4x - 12y - 1 = 0$ to the canonical form. Also, give the detailed description of the curve. (4+6+8)

5. (a) Discuss the convergences of the infinite series $x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \dots$
- (b) Using Taylor's series, expand $f(x) = x^3 + 8x^2 + 5x - 24$ in the powers of $(1+x)$. Also, find the value of $f(x)$.
- (c) Test the convergence of the following series :

$$\sum_{n=1}^{\infty} \frac{1}{\log_e(n+7)} \quad (7+7+4)$$

6. (a) The probability of a man hitting a target is $\frac{1}{4}$. If he fires 7 times, (i) find the probability of his hitting the target at least twice. (ii) How many times must he fire so that the probability of his hitting the target at least once is greater than $\frac{2}{3}$?
- (b) In a distribution exactly normal, 7% of the items are under 35 and 89% are under 63. What are the mean and variance of the distribution ?
- (c) Find the moment generating function of geometric distribution whose probability mass function is given by

$$P(X = x) = \begin{cases} q^x p; & x = 0, 1, 2, \dots, 0 < p \leq 1 \\ 0 & ; \text{ otherwise} \end{cases}$$

Also, find the variance of the distribution.

(8+5+5)

7. (a) Let X be a Poisson variate with parameter θ . Show that the only unbiased estimator of $e^{-(k+1)\theta}$, $k > 0$ is $T(X) = (-k)^x$ so that $T(X) > 0$, if x is even and $T(X) < 0$, if x is odd.
- (b) A twitcher sitting in a garden has spotted a number of birds belonging to 6 categories. The data is given below :

Birds Category	1	2	3	4	5	6
Frequency	6	7	17	13	6	5

Test whether or not the data is compatible with the assumption that this particular garden is visited by the birds belonging to these 6 categories in the proportion $1 : 1 : 3 : 2 : 1 : 1$ at 5% level of significance. [Given that $P(x^2 \geq 11.07) = 0.05$]

- (c) The coefficient of correlation between X and Y is 0.3. Their covariance is 9. The variance of X is 16. Find the standard deviation of Y series.

(6+8+4)

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