Sl. No.

B0.1-R5 : BASIC MATHEMATICS

NOTE :
1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Total Time : 3 Hours **Total Marks : 100**

1. (a) Sketch the function and determine its domain and range of $f(x) = \frac{x}{|x|}, x \neq 0$.

(b) Evaluate $\int t^4 \sqrt[3]{3-5t^5}$.

(c) Test the convergence of the series $\sum_{n=2}^{\infty} \frac{1}{(\log n)^n}$.

(d) Find the area inside the cardioid $r = 1 + \cos\theta$.

(e) Find the rank of the matrix $\begin{bmatrix} 6 & 0 & 4 & 4 \\ -2 & 14 & 18 & 18 \\ 14 & -14 & 0 & -10 \end{bmatrix}$.

(f) Graph the function $f(x)$ given by $f(x) = 2x^3 - 3x^2 - 12x - 12$ and find the relative extrema of $f(x)$.

(g) Evaluate definite integral of $\int_0^3 \left|1-x^2\right| dx$.

(7x4)

2. (a) Solve the systems of linear equation

$x - y + z = 1$

$2x + y - z = 2$

$5x - 2y + 2z = 5$

using Gauss-elimination method, if it is consistent.

(b) Find the angle between a diagonal of a cube and one of its edges.

(c) Show that the slope of every line tangent to the curve $y = \frac{1}{(1-2x)^3}$ is positive.

(6+6+6)

3. (a) Prove that the following function is discontinuous at $x=0$.

$$f(x)=\begin{cases}\dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}, & x\neq 0\\ 0, & x=0\end{cases}$$

(b) The circle $x^2+y^2=a^2$ is rotated about the x-axis to generate a sphere. Find its volume.

(c) A firm manufactures 5G smart phones and determines that after working t days, the efficiency, in number of phones produced per day, of most workers can be modelled by the function :

$N(t)=80-70e^{-0.13t}$

(i) Draw the graph of the function $N(t)$.

(ii) Find N'(t) and interpret this derivative in terms of rate of change.

(iii) What number of phones seems to determine where worker efficiency levels off ?

(6+6+6)

4. (a) Find the values of μ which satisfy the equation $A^{100}X=\mu X$, where $A=\begin{bmatrix}2 & 1 & -1\\ 0 & -2 & -2\\ 1 & 1 & 0\end{bmatrix}$.

(b) Find the orthogonal projection of $v=i+j+k$ on $b=2i+2j$ and then find the vector component of v orthogonal to b.

(c) Find the $\lim_{x\to+\infty}\sqrt{x^6+5x^3}-x^3$.

(8+6+4)

5. (a) Evaluate $\int\frac{2z\,dz}{\sqrt[3]{z^2+1}}$.

(b) Find the slopes of the tangent lines to the curve $y^2-x+1=0$ at points (2, -1) and (2, 1).

(c) Find the interval and radius of the convergence for the series $\sum_{n=0}^{\infty}\frac{(x-2)^n}{(n+1)3^n}$.

(5+6+7)

6. (a) Suppose that a particle moves on a coordinate line so that its velocity at time t is $v(t) = (t^2 - 2t\ )\text{m/s}$.

(i) Find the displacement of the particle during the time interval $0 \le t \le 3$.

(ii) Find the distance travelled by the particle during the time interval $0 \le t \le 3$.

(b) Show that there lies a point on the curve $f(x) = x(x+3)\, e^{-\frac{\pi}{2}}$, $-3 \le x \le 0$ where tangent drawn is parallel to the x-axis.

(c) Find the distance from the points (1, 1, 5) to the line L : $x = 1 + t$, $y = 3 - t$, $z = 2t$.

(6+6+6)

7. (a) Find the area of the region that is enclosed between the curves $y = x^2$ and $y = x + 6$.

(b) Show that the equation $x^2 - 4y^2 + 2x + 8y - 7 = 0$ represents the hyperbola. Find its centre, asymptotes and foci.

(c) Find all the eigen values and eigen vectors of the matrix $\begin{bmatrix} -3 & -7 & -5 \\ 2 & 4 & 3 \\ 1 & 2 & 2 \end{bmatrix}$.

(5+5+8)

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