No. of Printed Pages : **3**

Sl. No.

CE1.1-R4 : DIGITAL SIGNAL PROCESSING

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Total Time : 3 Hours **Total Marks : 100**

1. (a) Consider the system

$y(n) = T[x(n)] = x(n^2)$

(i) Determine if the system is time invariant. To clarify the result in part (i) assume that the signal

$$x(n) = \begin{cases} 1 & 0 \le n \le 3 \\ 0 & \text{elsewhere} \end{cases}$$

is applied into the system.

(ii) Sketch the signal $x(n)$

(iii) Determine and sketch the signal $y(n) = T[x(n)]$

(iv) Sketch the signal $y_2'(n) = y(n-2)$

(v) Determine and sketch the signal $y_2(n) = T[x(n-2)]$

(vi) Compare the signal $y_2(n)$ and $y(n-2)$.

(b) (i) Differentiate between the causal and non-causal system, stable and unstable system. Give the example of each.

(ii) Compute the convolution for the following pair of signals.

$$x(n) = \begin{cases} 1 & n = -2, 0, 1 \\ 2 & n = -1 \\ 0 & \text{elsewhere} \end{cases} \quad \text{and } h(n) = \delta(n) - \delta(n-1) + \delta(n-4) + \delta(n-5)$$

(c) Consider the linear constant-coefficient difference equation

$$y[n] - \frac{3}{4}\, y[n-1] + \frac{1}{8}\, y[n-2] = 2x[n-1]$$

Determine $y[n]$ for $n > 0$ when $x[n] = \delta[n]$ and $y[n] = 0$ for $n < 0$

(d) Explain the need of Z-transform. Find the system function H(z) and unit sample response h(n) of the system whose difference equation is described as under

$$y[n] = \frac{1}{2}\, y[n-1] + 2x[n]$$

where, $y[n]$ and $x[n]$ are the output and input of the system, respectively.

(e) Determine the Z-transform of following finite duration signals and draw their region of convergence (ROC).

(i) $X_1[n] = \{1, 2, 3, 4, 5\}$
$\uparrow$

(ii) $X_2[n] = \{1, 2, 3, 4, 5\}$
$\uparrow$

(f) What are the advantages and disadvantages of FIR Filters compared to IIR filters ?

(g) Differentiate between the Analog, digital and discrete time signal. Express the following discrete time signals using appropriate example.

(i) Even signal and Odd signal

(ii) Periodic and Non-periodic signal

(iii) Energy signal and power signal **(7x4)**

2. (a) Find the auto-correlation of the signal

$x[n] = a^n u[n]$, $0 < a < 1$

(b) Obtain direct form-I and direct form-II structures for the following system :

$Y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

(c) Determine the Fourier transform for the double exponential pulse whose function is given by $f(t) = e^{-a|t|}$. **(3x6)**

3. (a) Consider the filter $y(n) = 0.9y(n-1) + bx(n)$

(i) Determine b so that $|H(0)| = 1$.

(ii) Determine the frequency at which $|H(w)| = \frac{1}{\sqrt{2}}$.

(iii) Is the filter lowpass, bandpass or highpass ?

(b) Define the stability of system. A linear time-invariant system is characterized by the system function

$$H(z) = \frac{3 - 4z^{-1}}{1 - 3.5\,z^{-1} + 1.5\,z^{-2}}$$

Specify the ROC of H(z) and determine h(n) for the following conditions :

(i) The system is stable.

(ii) The system is causal.

(iii) The system anti-causal.

(c) Determine 4-point Discrete Fourier Transform (DFT) for the signal $x(t) = 1 + \cos(2\pi t)$ using radix-2 decimation in time FFT algorithm. **(3x6)**

4. (a) Determine the inverse z-transform of

$$X(z) = \frac{1}{1 - 1.5\,z^{-1} + 0.5\,z^{-2}} \text{ if}$$

(i) ROC : $|z| > 1$

(ii) ROC : $|z| < 0.5$

(iii) ROC : $0.5 < |z| < 1$

(b) Obtain direct cascade and parellel structures for the following system :

$Y(n) = -0.1y(n-1) + 0.2y(n-2) + 3x(n) + 3.6x(n-1) + 0.6x(n-2)$

(c) Explain the decimation and interpolation process with an example. **(3x6)**

5. (a) Consider a Linear Time Invariant (LTI) system with frequency response

$$H(e^{jw}) = e^{-j\left(w-\frac{\pi}{4}\right)} \frac{\left(1 + e^{-j(2w)} + 4e^{-j(4w)}\right)}{1 + \frac{1}{2e^{-j(2w)}}}, \quad -\pi < w < \pi$$

Determine the output $y[n]$ for all n if the input for all n is $x[n] = \cos\left(\frac{n\pi}{2}\right)$.

(b) Determine the step response of the causal system described by the difference equation $y(n) = y(n-1) + x(n)$.

(c) Perform the circular convolution of following two sequences

$x1(n) = \delta(n) + 2\delta(n-1) + 3\delta(n-3)$

$x2(n) = \delta(n) + 4\delta(n-1) + 2\delta(n-2) + 6\delta(n-3)$ **(3x6)**

6. (a) Given for low pass butterworth filter :

$Ap = -1$ db at 0.2π

$As = -15$fb at 0.3π

Calculate filter order N and pole location.

(b) Explain how frequency domain filters are used in image processing ? Explain its different types.

(c) Define Multi Rate Signal Processing. Describe applications of Multi Rate Signal Processing. **(3x6)**

7. (a) Design a digital Butterworth filter that satisfies the following constraint using bilinear transformation. Assume $T = 1$ second.

$0.9 \le |H(ejw)| \le 1$ $\qquad 0 \le w \le \frac{\pi}{2}$

$|H(ejw)| \le 0.2$ $\qquad 3\frac{\pi}{4} \le w \le \pi$

(b) Write a short note on TMS 320C50 Processor.

(c) Determine the variance of the round-off noise at the output of two cascade realization of the filter with system function

$H(z) = H_1(z)H_2(z)$

Where

$$H_1(z) = \frac{1}{1 - \frac{1}{2}z^{-1}} \text{ and } H_2(z) = \frac{1}{1 - \frac{1}{4}z^{-1}}$$ **(3x6)**

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