## C4-R4 : ADVANCED ALGORITHMS

## NOTE :

- 1. Answer question 1 and any FOUR questions from 2 to 7.
- 2. Parts of the same question should be answered together and in the same sequence.

## Total Time : 3 Hours

## Total Marks : 100

- **1.** (a) Give comparisons among Greedy, Divide and Conquer and Dynamic Programming algorithms and give the names of some algorithms that fall under them.
  - (b) Arrange the following in decreasing order of complexity : O(n!),  $O(n^2)$ ,  $O(2^n)$ , O(n logn).
  - (c) Give atleast any four applications of Breadth first search algorithm and show the BFS traversal for the given graph.
  - (d) Find the time complexity of Prim's algorithm and Krushkal's algorithm. Give one example of each.
  - (e) Differentiate between NP, CoNP, NPC and NP-hard problems.
  - (f) Write a recursive program for Binary search algorithm. Give its recursion relation and worst case time complexity.
  - (g) Differentiate between Radix Sort and Bucket Sort.

(7x4)

- **2.** (a) Suppose that all the characters in the pattern P are different. Show how to accelerate NAÏVE STRING MATCHER to run in time O(n) on an n-character text T.
  - (b) Using the KMP Algorithm find whether 'P' occurs in  $\infty$ .

T : b a c b a b a b a b a c a c a, P : a b a b a c a

(c) Consider 5 items along with their respective weights and values :

 $I = < I_1, I_2, I_3, I_4, I_5 >$ w = < 5, 10, 15, 25, 30 > v = < 50, 20, 100, 150, 240 > The capacity of knapsack W = 50. Find the solution to the fractional knapsack

problem.

(5+8+5)

- 3. (a) Another way to perform topological sorting on a directed acyclic graph G = (V,E) is to repeatedly find a vertex of in degree zero, output it, and remove it and all of its outgoing edges from the graph. Explain how to implement this idea so that it runs in time O(V + E). What happens to this algorithm if G has cycles ?
  - (b) Give and solve the recursion relation for solving the Tower of Hanoi problem with one example considering 3 disks.
  - (c) Illustrate the operation of BUCKET SORT on the array.
    A = < 0.36, 0.12, 0.22, 0.85, 0.56, 0.74, 0.33 >

(6+8+4)

- **4.** (a) What is Triangle-inequality in approximate algorithms ? List out some of the problems that can have solution using approximate algorithm.
  - (b) Write the sorting algorithms of the following :
    - (i) Selection sort
    - (ii) Shell sort
    - (iii) Radix sort
  - (c) Using the recurrence relation check that  $a_n = 2^n + 1$  is a solution to the  $a_n = 2a_{n-1} 1$  with  $a_1 = 3$ . (6+9+3)
- 5. (a) Differentiate between 0/1 knapsack problem and Fractional problem.
  - (b) Construct the minimum spanning tree for the graph given below using Prims Algorithm.



- (c) What is the running time of heap sort on an array A of length n that is sorted in increasing order ? What about decreasing order ? (6+6+6)
- **6.** (a) Define Euler Path and Euler cycle and prove that if Euler path exists or not in the graph shown below :



- (b) Determine whether the relation R on a set A is reflexive, irreflexive, symmetric, asymmetric, antisymmetric or transitive.
  - (i)  $A = set of all positive integers, a R b iff <math>a b \le 2$ .
- (c) Prove that  $\log_2 n$  is  $O(n^a)$  for any a > 0.

(5+7+6)

7. (a) Find the vertex cover of the given graph.



(b) "The recurrence  $T(n) = 7T\left(\frac{n}{2}\right) + n^2$  describes the running time of an algorithm A.

A competing algorithm A' has a running time of  $T'(n) = T'\left(\frac{n}{2}\right) + n^2$ . What is the

largest integer value for 'a' such that A' is asymptotically faster than A  $?^{\prime\prime}$ 

(c) Consider following 'for' loops, calculate the total computation time of the following:

(7+6+5)

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