

### C3-R4: MATHEMATICAL METHODS FOR COMPUTING

#### NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) In a bolt factory, machines A, B and C manufacture 25, 35 and 40 percent of the total of their output 5, 4 and 2 percent are defective. A bolt is drawn at random and is found to be defective. What is the probability that it was manufactured by the machine B. Solve using Baye's theorem.
- b) If a random variable has a Poisson distribution such that  $P(1) = P(2)$ , find  $P(4)$ .  
(use  $e^{-2} = 0.1353$ )
- c) The density function for a continuous random variable  $x$  is given by

$$f(x) = \begin{cases} \frac{1}{2} \sin x, & 0 \leq x \leq \pi \\ 0, & \text{otherwise} \end{cases}$$

Find the mean and variance.

- d) Let  $X$  be a random variable with probability distribution

$$\begin{array}{cccc} X & : & 0 & 1 & 2 & 3 \\ f(X) & : & \frac{1}{3} & \frac{1}{2} & 0 & \frac{1}{6} \end{array}$$

Find the expected value of  $(X - 1)^2$ .

- e) Write the dual of the following linear programming problem

Maximize  $Z = 5x_1 + 10x_2$

Subject to

$$2x_1 - 3x_2 \leq 7$$

$$x_1 + 2x_2 = 4$$

$$x_1, x_2 \geq 0$$

- f) Find the Laplace transform of  $\frac{\sin 2t}{t}$ .
- g) Find the probability distribution of the number of successes in two tosses of a fair die, where a success is defined as a number greater than 4.

(7x4)

2.

- a) Solve the following linear programming problem by the simplex method:

Maximize  $Z = 3x_1 + 2x_2$

Subject to the constraints:

$$x_1 + x_2 \leq 4$$

$$x_1 - x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

b) Find the Fourier series of the function

$$f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$$

and hence show that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

**(9+9)**

**3.**

a) There is congestion on the platform of a railway station. The trains arrive at the rate of 30 trains per day. The waiting time for any train to hump is exponentially distributed with an average of 36 minutes. Calculate the following:

- i) The mean queue size (average number of trains in the queue).
- ii) The probability that queue size exceeds 9.

b) Solve the following integer linear programming problem by branch and bound technique

Maximize  $Z = x_1 + x_2$

Subject to

$$3x_1 + 2x_2 \leq 12$$

$$x_2 \leq 2$$

$$x_1, x_2 \geq 0 \text{ and both are integers.}$$

**(9+9)**

**4.**

a) Find the inverse Laplace transform of  $\frac{s+4}{s(s-1)(s^2+4)}$ .

b) Suppose there are two market products of brand A and B, respectively. Let each of these two brands have exactly 50% of the total market in the same period and let the market be of a fixed size. The transition matrix is given below:

$$\begin{array}{c} \text{To} \\ \text{From} \end{array} \begin{array}{cc} \text{A} & \text{B} \\ \text{A} & \begin{bmatrix} 0.9 & 0.1 \end{bmatrix} \\ \text{B} & \begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \end{array}$$

If the initial market share breakdown is 50% for each brand, then determine their market shares in the steady state.

**(9+9)**

**5.**

a) Given the probability distribution of  $X$ :

$X$	0	1	2	3	4	5	6	7
$f(X)$	0	$C$	$2C$	$3C$	$2C$	$C^2$	$2C^2$	$7C^2 + C$

- i) Find  $C$ .
- ii) Evaluate  $P(X \geq 5)$  and  $P(X < 3)$ .

- b) A box containing  $2^n$  tickets among which  ${}^n C_4$  bear the number  $i, (i = 0, 1, 2, \dots, n)$ . A group of  $m$  tickets is drawn, what is the expectation of the sum of numbers. (8+10)

6.

- a) A source without memory has six characteristics with the following probabilities of transmission

$A$	$B$	$C$	$D$	$E$	$F$
$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{12}$	$\frac{1}{12}$

Derive the Shannon-Fano encoding procedure to obtain uniquely decodable code to the above message ensemble and find its average length.

- b) Use dynamic programming technique to solve the following problem

Maximize  $y_1, y_2, y_3$

Subject to

$$y_1 + y_2 + y_3 = 5, \quad y_1, y_2, y_3 \geq 0$$

(9+9)

7.

- a) Arrivals at a telephone booth are considered to be following Poisson law of distribution with an average time of 10 minutes between one arrival and the next. Length of a phone call is assumed to be distributed exponentially with mean 3 minutes.

i) What is the probability that a person arriving at the booth will have to wait?

ii) What is the average length of queue that forms from time to time?

- b) A transmitter has an alphabet consisting of five letters  $\{x_1, x_2, x_3, x_4, x_5\}$  and the receiver has an alphabet consisting of four letters  $\{y_1, y_2, y_3, y_4\}$ . The joints probabilities for the communication are given below:

	$y_1$	$y_2$	$y_3$	$y_4$
$x_1$	0.25	0	0	0
$x_2$	0.10	0.30	0	0
$x_3$	0	0.05	0.10	0
$x_4$	0	0	0.05	0.10
$x_5$	0	0	0.05	0

Determine the Marginal, Conditional and Joint entropies for this channel (Assume  $0 \log 0 \cong 0$ ).

(8+10)