No. of Printed Pages : **4**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same questions should be answered together and in the same sequence.**

Total Time : 3 Hours **Total Marks : 100**

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1. (a) Let X have the discontinuous cdf

$$F_x(x) = 0 \quad \text{if} \quad x < 0,$$

$$= \frac{x}{2} \quad \text{if} \quad 0 \le x < 1,$$

$$= 1 \quad \text{if} \quad 1 \le x$$

Find $P\left(-1 \le X \le \frac{1}{2}\right)$ and $P(X=1)$.

(b) Let $\{X_t : t=1, 2, \ldots\}$ be a Markov chain with the transition matrix

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 04 & 0 & 0.6 \\ 0 & 0.8 & 0.2 \end{bmatrix}$$

Determine $P = \left(\frac{X_2 = 3}{x_0 = 1}\right)$.

(c) Let X be the number of products manufactured in a factory during a week. Suppose $E(X) = 100$, and $Var(X = 400$. Find an upper bound on the probability that this week's production will be at least 120.

(d) Let X be a standard normal variable, and Y be independent of X with $P(Y = 1) = \frac{1}{2} = P(Y = -1)$. Let $Z = YX$. Show that X and Z are uncorrelated, however they are dependent.

(e) Find $L^{-1}\left\{\frac{1}{\sqrt{2s+3}}\right\}$, L^{-1} is the Laplace inverse.

(f) The joint density function of X, Y is

$$f(x, y) = \frac{1}{y} e^{-(y+x/y)}, \ 0 < x, y < \infty$$

Verify that the preceding is a joint density function.

(g) Suppose we have two discrete random variables X and Y, with the following joint probability distribution :

$P(X=x_i . Y=y_j$	$Y=0$	$Y=1$	$Y=2$
$X=0$	0.1	0.2	0.1
$X=1$	0.2	0.1	0.0
$X=2$	0.1	0.1	0.4

Find H(X). **(7x4)**

2. (a) Let X and Y have joined pdf

$f(x, y)=24xy$ if $0 < x_1, 0 < y , x+y < 1$, and

$=0$ elsewhere

Find the regression equation of Y on X, that is, the conditional expectation $E(Y|X=x)$

(b) Find $L^{-1}\left\{\frac{1}{((s+1)(s^2+1))}\right\}$, L^{-1} is the Laplace inverse. **(12+6)**

3. (a) Let $(X, Y) \sim BVN\left(0,\ 0,\ 2,\ \frac{\sqrt{17}}{2},\ \rho\right)$, i.e. bivariate normal distribution with means,

$E(X)=0$, $E(Y)=0$, $var(X)=2$, $var(Y)=\frac{\sqrt{17}}{2}$, and ρ is the correlation coefficient.

Suppose $E(XY)=2$, and $Z=2X-3Y$. Determine the pdf of Z.

(b) Let X and Y have joint pdf : n

$f(x, y)=e^{-y}(1-e^{-x})$, if $0 < x < y, 0 < y < \propto$

$=e^{-x}(1-e^{-y})$, if $0 < y < x, 0 < x < \propto$

Determine the marginal pdf's of X and Y. Are X and Y independent ?

(8+10)

4. (a) Use branch and bound method to solve the following integer LPP :

Max $z=3x_1+4x_2$

Subject to

$7x_1+16x_2 \le 52$

$3x_1-2x_2 \le 9$

$x_1, x_2 \geqslant 0$

x_1 and x_2 are integer.

(b) Find the Fourier series to the function, f on the interval $0 \le x \le \pi$ given by

$$f(x) = x \quad \text{if } 0 \le x \le \frac{\pi}{2}, \text{ and}$$

$$= \pi - x \quad \text{if } \frac{\pi}{2} \le x \le \pi$$

(10+8)

5. (a) Ninety-eight percent of all babies survive delivery. However, 45 percent of all births involve Cesarean (C) sections, and the baby survives 46 percent of the time only if a C section is performed. If a randomly chosen pregnant woman does not have a C section, what is the probability that her baby survives ?

(b) Let X and Y represent the performance of two students in two different subjects, Math and English. Each of the subjects has three grades, A, B and C. The joint probability distribution is given by :

	Math (X)		
English (Y) :	A	B	C
A	0.2	0.1	0.1
B	0.1	0.2	0.1
C	0.1	0.1	0.1

Find the mutual information between the Math and English grades.

(10+8)

6. (a) A group of airtel subscribers is observed continuously during a 60-minute busy-hour period. During this time, they make 20 call, with the total conversation time being 800 seconds. Compute the call arrival rate and the traffic intensity.

(b) Assume that an inverter system is in one of the three states: On, Off or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at 12 P.M. each day, it is believed that the system approximately behaves like a homogenous Markov chain with the transition probability matrix:

$$P = \begin{bmatrix} 0.4 & 0.3 & 0.3 \\ 0.2 & 0.6 & 0.2 \\ 0.5 & 0 & 0.5 \end{bmatrix}$$

Prove that the chain is irreducible, and determine the steady-state probabilities.

(8+10)

7. (a) Consider the problem

Min $f(x) = x_1 + x_2$

subject to

$x_1 + 3x_2 \leq 12$

$x_1 - x_2 \leq 1$

$2x_1 - x_2 \leq 4$

$2x_1 + x_2 \leq 8$

$x_1, x_2 \geqslant 0$

Does the solution through the simplex algorithm converge ?

(b) Solve the following non-linear problem

Min $\left(x_1^2 + 2x_2^2\right)$

subject to

$x_1^2 + 2x_2^2 \leq 5$

$2x_1 - 2x_2 = 1$

using Kuhn-tucker Conditions. **(10+8)**

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