No. of Printed Pages : **3**

Sl. No.

# C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES

**NOTE :**

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

**Total Time : 3 Hours** **Total Marks : 100**

**1.** (a) Show that the matrix $\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$ is a solution of the equation $A^2 - 5A + 7I = O$, Where I is $2 \times 2$ identity matrix.

(b) Find $\frac{dy}{dx}$ if $y = \cos(x^x)$

(c) Evaluate : $\int \sqrt{1 - \sin x}\, dx$.

(d) Verify the mean value theorem for $f(x) = 2x^2 - 7x + 10$ in [2, 5].

(e) Find the value of P for which the Vector $\vec{A} = 3\hat{i} - \hat{j} + 4\hat{k}$ and Vector $\vec{B} = P\hat{i} + 3\hat{j} + \hat{k}$ are perpendicular.

(f) A fair die is rolled. Consider the following events A = {2, 4, 6}, B = {4, 5} and C = {3, 4, 5, 6}. Find P(A ∪ BIC) and P(A ∩ BIC)

(g) Find the Eigen values of the matrix $A = \begin{bmatrix} 3 & 2 \\ -1 & 0 \end{bmatrix}$ **(7x4)**

**2.** (a) Examine the continuity of the function $f(x)$ defined by

$$f(x) = \begin{cases} 5x - 4, & \text{if } 0 < x \le 1 \\ 4x^3 - 3x, & \text{if } 1 < x < 2 \end{cases} \quad \text{At } x = 1$$

(b) Find the rank of the matrix.

$$\begin{bmatrix} 2 & 3 & 4 \\ 3 & 1 & 2 \\ -1 & 2 & 2 \end{bmatrix}$$

(c) Find $\frac{dy}{dx}$ of following.

(i) $y = \sin(e^{\tan x})$ (ii) $y = \sec^2(\tan^{-1} x)$ **(4+5+9)**

**3.** (a) Find the probability distribution of number of doublets in three throws of a pair of dice.

(b) Prove that $\begin{bmatrix} b+c & a+b & a \\ c+a & b+c & b \\ a+b & c+a & c \end{bmatrix} = a^3 + b^3 + c^3 - 3abc$

(c) Evaluate : $\int \frac{x}{(x-2)(x-1)^2}\, dx$ **(6+6+6)**

**4.** (a) Show the system of equation $x+2y+3z=11$; $x-2y+3z=3$ and $x+2y-3z=-1$ is consistent and find the solution.

(b) Evaluate : $\lim_{x\to 0} (1+x)^{1/x}$

(c) Sketch the graph of $y=|x+3|$ and evaluate $\int_{-6}^{0} |x+3|\, dx$ **(6+6+6)**

**5.** (a) Discuss the convergences of following series.

$$1 + \frac{2^1}{2!} + \frac{3^2}{3!} + \frac{4^3}{4!} + \ldots \infty$$

(b) Find regression equation of β on α and estimate β when $\alpha = 55$ from the following data :

| α | 40 | 50 | 38 | 60 | 65 | 50 | 35 |
|---|---|---|---|---|---|---|---|
| β | 38 | 60 | 55 | 70 | 60 | 48 | 30 |

(c) A sample of 144 transistors manufactured by a company is found to have an average life of 1450 days with a standard deviation of 100 days. Establish 90% confidence limits within which the mean lifetime of a transistor is expected to lie. The significant value at 10% is $Z\alpha/2 = 1.645$. **(6+6+6)**

**6.** (a) Use Cramer's rule to solve the following system of equations.

$x-y-z=1$, $y-z-x=1$, $z-x-y=1$

(b) Find the equation of a circle which passes through $(2, -3)$ and $(-4, 5)$ and having center on the line $4x+3y+1=0$.

(c) Find the angle between the vectors $\vec{A} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{B} = \hat{i} - \hat{j} + \hat{k}$. **(6+6+6)**

**7.** (a) In a factory, machines A, B and C produce 60%, 30% and 10% of certain items respectively. 1%, 2% and 3% of the item produced respectively by A, B and C are found to be defective. A certain item is picked up at random from the total production and found to be defective. Find the probability that defective item is produced by machine A.

(b) Prove that $\int_0^{\frac{\pi}{4}} \log_e(1 + \tan x) \cdot dx = \frac{\pi}{8} \cdot \log_e 2$

(c) Test the convergence of series :

$$\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \ldots \infty$$ **(6+6+6)**

**- o O o -**