C0-R4.B1: ELEMENTS OF MATHEMETICAL SCIENCES

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let $\vec{a} = 2i - j + k, \vec{b} = i - 3j - 5k$ and $\vec{c} = 3i - 4j - 4k$ be three vectors. Find $\vec{a}x(\vec{b}x\vec{c})$.

b) Find the equation of the line through the point of intersection of 5x-3y = 1 & 2x + 3y = 23 and perpendicular to the line whose equation is 5x-3y = 1.

c) Let $f(x) = \begin{vmatrix} 1/\sqrt{2} & \sin x & 1 \\ 1/\sqrt{2} & \cos x & x \\ 1 & 1 & x^2 \end{vmatrix}$

Then what is the value of f(π/4)?

d) If α and β are roots of the quadratic equation $ax^2+bx+c=0$, $a \neq o$, then formulate a quadratic equation whose roots are $(\alpha-\beta)^2$ and $(\alpha+\beta)^2$.

e) In how many ways we can choose 2 persons of different genders out of 6 men and 5 women?

f) Evaluate $(\cos^2 x - \sin^2 x)/(\sin x \cos x)\, dx$.

g) Evaluate $\lim_{x \to 2a} \dfrac{\sqrt{x-2a} + (\sqrt{x} - \sqrt{2a})}{\sqrt{x^2 - 4a^2}}$.

(7x4)

2.

a) Find a matrix A satisfying the equation

$$\begin{bmatrix} 2 & 1 \\ 3 & 3 \end{bmatrix} A \begin{bmatrix} 5 & 3 \\ 3 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

b) Solve the following set of equations using Cramer's rule

2x - y +3z =9

X + y +z = 6

X - y + z =2

c) If $y \sin^2 x = 2x$ then find $\dfrac{dy}{dx}$.

(6+6+6)

3.

a) Evaluate the determinant

$$\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix}$$

b) Determine the point on the curve $y^2 = 4x$ at which tangent to the curve is parallel to the line y = x.

c) Determine first three terms in the expansion of $(1 - 2x^3)^{11/2}$.

(6+6+6)

4.

a) Determine the maximum value of $y = 2\cos 2x - \cos 4x$ when $0 \le x \le \pi/2$.

b) Find the eigen values and eigen vectors of the matrix

$$\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$$

(9+9)

5.

a) Determine the rank of the following matrix A. Does the inverse of A exist? If yes find it, if not why?

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}$$

b) A random variable X has the following probability function:

X:	0	1	2	3	4	5	6	7
P(x):	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

Evaluate $P(X \ge 6)$ and $P(X < 4)$. Also find expectation E(X).

(9+9)

6.

a) Let A and B be two events such that P(A) = 3/4 and P(B) = 5/8, show that

i) $P(A \cup B) \ge 3/4$

ii) $3/8 \le P(A \cap B) \le 5/8$

b) Let P(6, 3) be a point on the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. If the normal at the point P intersects the x-axis at (9, 0) then find eccentricity and length of latus rectum of the hyperbola.

(9+9)

7.

a) A set of 5 coins is tossed 3200 times and the number of heads appearing each time is noted. The results are given below:

No of head:	0	1	2	3	4	5
Freq	80	570	1100	900	500	50

Test the hypothesis that the coins are unbiased.

b) Find the limit when $n \to \infty$ of the series

$$\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \ldots + \frac{1}{3n}$$

(10+8)