No. of Printed Pages : **2**

Sl. No.

B32-R4 : DISCRETE STRUCTURE

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Total Time : 3 Hours **Total Marks : 100**

1. (a) Show that $f(x, y) = x^y$ is primitive recursive function.

(b) What are the truth values of the propositions R(1, 2, 3) and R(0, 0, 1) ?

(c) How many different Boolean function of degree n are there ?

(d) Consider the following algorithm segment :

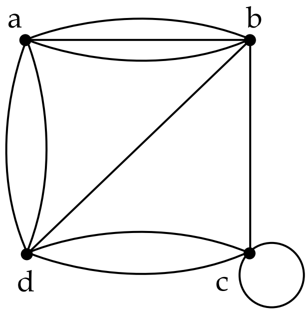
```
i:=1, sum:=0
While(i<10)
sum:=sum+i
i:=i+1
end while
```

what is the outcome of the loop ?

(e) How many edges are there in a graph with 10 vertices each of degree six ?

(f) Each user on a computer system has a password, which is six to eight characters long, where each character is an uppercase letter or a digit. Each password must contain at least one digit. How many possible passwords are there ?

(g) Use an adjacency matrix to represent the pseudo graph shown in below figure.



(4x7)

2. (a) In a survey of 60 people, it was found that 25 people read newspaper H, 26 read newspaper T, 26 read newspaper I, 9 read both H and I, 11 read both H and T, 8 read both T and I, 3 read all three newspapers. Find

(i) the number of people who read at least one of the newspapers.

(ii) the number of people who read exactly one newspaper.

(b) A committee of three individuals decides issues for an organization. Each individual votes either yes or no for each proposal that arises. A proposal is passed if it receives at least two yes votes. Design a circuit that determines whether a proposal passes.

(c) Let a and b be two positive integers. Prove that gcd(a, b). lcm(a, b) = ab.

(6+6+6)

3. (a) Find the inverse of the permutation :

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$$

(b) Give a big-O estimate for $f(n) = 3n \log(n!) + (n^2 + 3) \log n$, where n is a positive integer.

(c) Show that among any n + 1 positive integers not exceeding 2n there must be an integer that divides one of the other integers. **(6+6+6)**

4. (a) Use the insertion sort to put the elements of the list 3, 2, 4, 1, 5 in increasing order.

(b) Prove that the set {0, 1, 2, 3, 4} is a finite abelian group of order 5 under addition modulo 5 as composition.

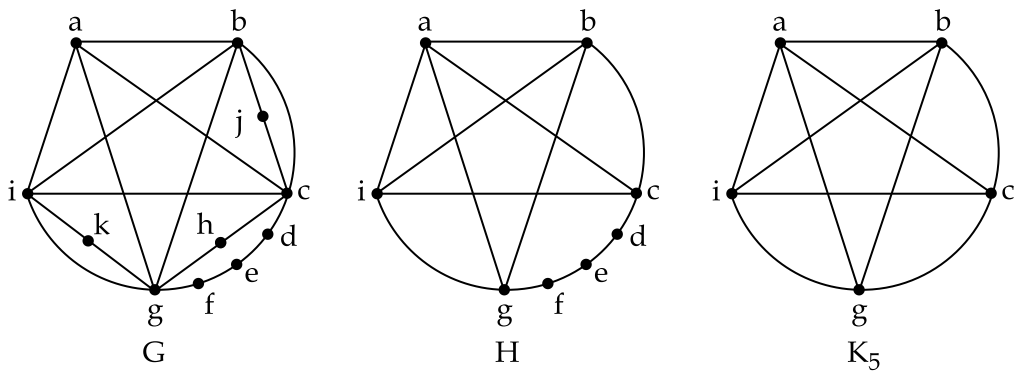
(c) Find the Fibonacci numbers f_2, f_3, f_4, f_5 and f_6. **(6+6+6)**

5. (a) An important element in many electronic devices is a *unit-delay machine*, which produces as output the input string delayed by a specified amount of time. How can a finite-state machine be constructed that delays an input string by one unit of time, that is, produces as output the bit string $0\ x_1 x_2 \ldots x_{\{k-1\}}$ given the input bit string $x_1 x_2 \ldots x_k$?

(b) What is the truth value of $\forall x (x^2 \geq x)$ if the domain consists of all real numbers ? What is the truth value of this statement if the domain consists of all integers ?

(c) Find Karnaugh Map and simplify the expression: X = A'B'C' + A'BC' + ABC' + AB'C. **(5+8+5)**

6. (a) Determine whether the graph G shown in below figure is planar.



(b) Find a Turing machine that recognizes the set $\{0^n 1^n \mid n \geq 1\}$.

(c) Show that $\neg(p \vee (\neg p \wedge q))$ and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences. **(6+6+6)**

7. (a) Prove that a simple graph is connected if and only if it has a spanning tree.

(b) Show that the sequence {2, 3, 4, 5, …., 2 + n, ….} for n ≥ 0 satisfies the recurrence relation $a_k = 2a_{k-1} - a_{k-2}$, $k \geq 2$.

(10+8)

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