No. of Printed Pages : **3**

Sl. No.

B1.2-R5 : DISCRETE STRUCTURES

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Total Time : 3 Hours **Total Marks : 100**

1. (a) It is desired to generate the sequence (s) of seven digits. Out of seven, three of them are capital letters (A to Z) and four of them are numbers (0 to 9). Numbers and letters can be repeated in the sequence. You need to follow the following pattern while generating the seven digits sequences : the 1^{st} digit of the sequence must be a number, the next three digits are letters, and the last three digits are numbers. How many sequences could be created with 3 letters and 4 numbers ?

(b) Considering that digits can be repeated, how many 3 digit numbers which are greater than 300 can be formed from the digits 2, 3, 4, 5, 7, and 9.

(c) Suppose a person is working at a library. He must put 5 books on a shelf in any order. How many different ways can he order the books on the shelf ?

(d) Discuss different types of quantifiers in predicate logic.

(e) Justify, which one of the following statement is proposition ?

(i) It rained Yesterday

(ii) What is your name

(f) You have been given the dimensions (row $\times$ col) of four matrices as follows: A (40×20), B (20×30), C (30×10), and D (10×30). Applying the Dynamic Programming based approach for matrix chain multiplication, compute the minimum number of multiplications needed to perform the multiplication of all four matrices, i.e. $A \times B \times C \times D$.

(g) We can express algorithmic complexity using the big-O notation. Define the big-O notation. **(7x4)**

2. (a) The Longest Common Subsequence (LCS) problem is one where, it is required to find the longest sequence (might not be contiguous) in common between given two sequences. Often it is solved using Dynamic Programming (DP). Present the DP based recurrence relation to solve the problem of LCS. Using the presented recurrence relation and depicting the initialization of the memorization table and the final memorization table, explain, how you will obtain the longest common subsequence between following two sequences (X and Y):

X = A C A D B

Y = C B D A

(b) In context of Dynamic Programming (DP), briefly discuss the overlapping sub-problems. Explain the concept of overlapping sub-problem by finding the N^{th} Fibonacci number and justify that storing the answer for the overlapping subproblems reduces the time complexity. **(9+9)**

3. (a) Mention the best case, average case, and worst case time complexities of following sorting algorithms: Selection sort, Bubble sort, Insertion sort, Heap sort, Quick sort, and Merge sort.

(b) Why sorted data makes the searching more efficient ? Besides efficient searching, mention examples where sorted data is helpful.

(c) What is the significance of bisect (array bisection algorithm) module in Python. List down the functions provided in the bisect module. Further, write Python code to demonstrate the working of binary search using functions in the bisect module. **(6+4+8)**

4. (a) In graph theory, a planar graph is a graph that can be embedded in the plane, i.e., it can be drawn on the plane in such a way that no edges cross each other. Considering that there are no self-loops and parallel edges, give proper argument about planarity / non-planarity of following graphs:

(i) A graph, G with 6 vertices and 10 edges

(ii) A graph, G with 6 vertices and 9 edges

(iii) A graph, G with 6 vertices as A, B, C, D, E, and F and following edges (represented by mentioning two end vertices of an edge): AD, AE, AF, BD, BE, BF, CD, CE, and CF.

(b) Apply Dijkstra's algorithm to find out shortest path to all the vertices from vertex 'A' in the graph given in Fig. 1.

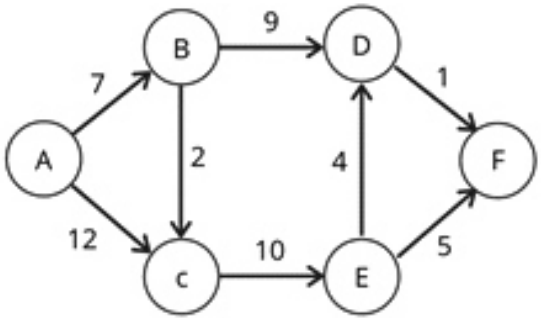

Fig. 1 **(9+9)**

5. (a) Depth first search (DFS) is one of the traversal schemes in Graph. An undirected graph is given in Fig. 2 which needs to be traversed using DFS. If the traversal is started from vertex A, justify, which one of the following can and cannot be the order of the traversed vertices (vertices are listed in the order they are visited first).

(i) A B E H G C F D

(ii) A B E D C F G H

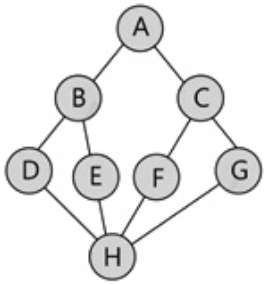

Fig. 2

(b) Through examples, explain the difference between permutation and combination. **(10+8)**

6. (a) In context of graphs, briefly discuss Walk, Trails, Circuits, Path, and Cycle. Further, in the graph given in Fig. 3, which of the following sequence of the vertices determines walks. For those sequences which are walk, you need to also determine whether it is a cycle, path, circuit, or trail.

(i) A, B, G, F, C, D

(ii) B, G, F, C, B, G, A

(iii) C, E, F, C

(iv) C, E, F, C, E

(v) A, B, F, A

(vi) F, D, E, C, B

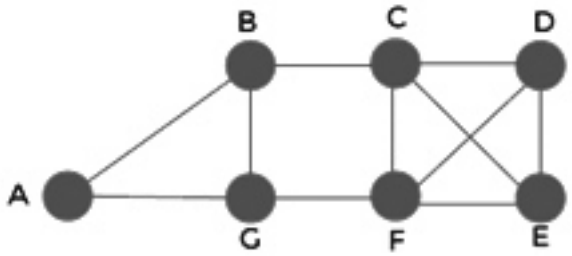

Fig. 3

(b) A box contains 6 red, 8 green, 10 blue, 12 yellow and 15 white balls. What is the minimum no. of balls we have to choose randomly from the box to ensure that we get 9 balls of same color ?

(12+6)

7. (a) Let $A = \{1, 2, 3, 4\}$ and $R = \{(1, 1), (1, 3), (2, 2), (2, 4), (3, 1), (3, 3), (4, 2), (4, 4)\}$. Show that R is an Equivalence Relation.

(b) In a group of 100 students, 72 students can speak English and 43 students can speak Hindi. Based on these data, answer the following questions:

(i) Find the number of students who can speak English only.

(ii) Find the number of students who can speak Hindi only.

(iii) Find the number of students who can speak both English and Hindi.

(c) Prove that the statement $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tautology.

(4+9+5)

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