

B0.1-R5 : BASIC MATHEMATICS**NOTE :**

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Total Time : 3 Hours**Total Marks : 100**

1. (a) Consider the power series as

$$\sum_{n=0}^{\infty} 2^n x^n$$

Find the function f represented by this series. Determine the interval of convergence of the series.

- (b) Using an example, discuss the triangle property of determinants.
- (c) Discuss the properties of symmetric matrix and skew-symmetric matrix.
- (d) Give the parametric equation of a straight line passing through (x_1, y_1) and making an angle θ with the positive X-axis.
- (e) Using differential calculus find the derivative of $x(\sin x)$.
- (f) How differential calculus is different than integral calculus ? Briefly discuss.
- (g) Evaluate the following indefinite integral :

$$\int x^4 + 3x - 9 \, dx \quad (7 \times 4)$$

2. (a) What do you mean by a tangent to a circle? Write the steps to find Tangent and Normal to a Circle.
- (b) What are Conic Sections ? List down different types of conic sections.
- (c) Convert the polar coordinate $\left(4, \frac{\pi}{2}\right)$ to a cartesian coordinate point. (6+6+6)

3. (a) Find the two numbers whose arithmetic mean is 25 and geometric mean is 20.
- (b) Solve the following linear equations using Gauss elimination method:

$$\begin{aligned} X + Y + Z &= 3 \\ X + 2Y + 3Z &= 0 \\ X + 3Y + 2Z &= 3 \end{aligned}$$
(9+9)

4. (a) Find the Taylor polynomials, p_0, p_1, p_2 and p_3 for $f(x) = \ln x$ at $x = 1$.
- (b) Find the area of a parallelogram whose adjacent sides (a and b) are given as follows :

$$\begin{aligned} a &= 4i + 2j - 3k \\ b &= 2i + j - 4k \end{aligned}$$

- (c) What is successive differentiation ? Find out $\frac{d^2y}{dx^2}$ and $\frac{d^3y}{dx^3}$ where y is given as follows :

$$y = 1 + \frac{x}{1} + \frac{x^2}{1 \times 2} + \frac{x^3}{1 \times 2 \times 3} + \frac{x^4}{1 \times 2 \times 3 \times 4} \quad (6+6+6)$$

5. (a) Briefly discuss the steps to check, if a function is continuous on an interval. Further, find out, for which one of the following intervals, the function, $f(x) = \sqrt{x-4}$ is continuous.
- (i) Interval [4, 7)
(ii) Interval [0, 7)
- (b) What is L'Hospital's rule ? When it can or cannot be used ? Using L'Hospital's rule, evaluate following expressions :

(i) $\lim_{x \rightarrow 0} \frac{(2\sin x - \sin 2x)}{(x - \sin x)}$

(ii) $\lim_{x \rightarrow 0} \frac{\sin 3x}{\sin 4x}$ (9+9)

6. (a) Determine all the numbers c which satisfy the conclusions of the Mean Value Theorem for the following function :

$$f(x) = x^3 + 2x^2 - x \text{ on } [-1, 2]$$

- (b) Evaluate the following integrals using method of substitution :

(i) $\int \frac{\cos(\sqrt{x})}{\sqrt{x}} dx$

(ii) $\int 2x^3 \sqrt{x^2 + 1} dx$ (9+9)

7. (a) Find the area bounded by curves $(x-1)^2 + y^2 = 1$ and $x^2 + y^2 = 1$.
(b) In context of Integration by Parts, briefly discuss LIATE or ILATE rule. Also, evaluate the following integral using the method of integration by parts :

$$\int x \cos x \quad (10+8)$$

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