No. of Printed Pages : **3**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :

1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

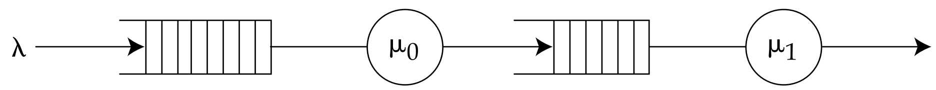
1. (a) Suppose that the population of a certain city is 40% male and 60% female. Suppose also that 50% of the males and 30% of the females smoke. Find the probability that a smoker is male.

(b) A markov chain $X_1, X_2, \ldots$ on states 0, 1, 2 has the transition probability matrix

$$P = \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.9 & 0.1 & 0 \\ 0.1 & 0.8 & 0.1 \end{bmatrix} \end{array}$$

and initial distribution $p_0 = \Pr\{X_0 = 0\} = 0.3$, $p_1 = \Pr\{X_0 = 1\} = 0.4$ and $p_2 = \Pr\{X_0 = 2\} = 0.3$. Determine $\Pr\{X_0 = 0, X_1 = 1, X_2 = 2\}$

(c) A repair facility shared by a large number of machines has two sequential stations with respective rates one per hour and two per hour. The cumulative failure rate of all the machines is 0.5 per hour. Assuming that the system behavior may be approximated by the two-stage tandem queue



shown in Figure above, determine the average repair time.

(d) Let random variables X and Y have joint probability function

$p(x_i, y_j) = \Pr\{X = x_i, Y = y_j\} = p_{ij}$, $i = 1, 2, \ldots, M$, $j = 1, 2, \ldots, L$. Show that :

$H(X, Y) = H(X) + H(Y|X)$

(e) Find the dual of following linear programming problem :

$$\text{Max } Z = 2x_1 + 3x_2 + x_3$$

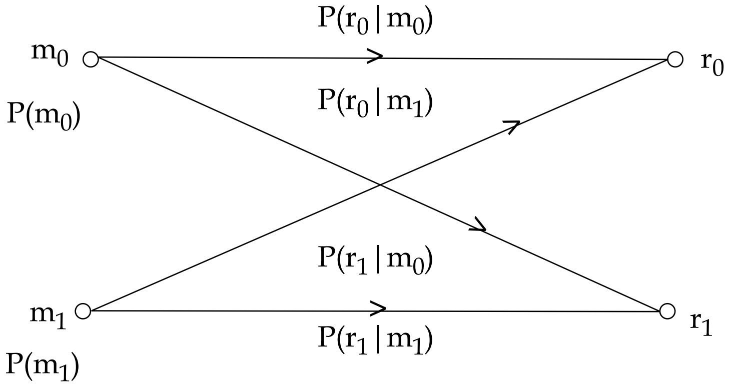
Subject to
$$4x_1 + 3x_2 + x_3 = 6,$$
$$x_1 + 2x_2 + 5x_3 = 4$$
$$x_1, x_2, x_3 \geq 0$$

(f) Find the Laplace Transform of $f(t) = \sinh at$

(g) Write an algorithm to simulate random variable X, from the distribution function given by $F(x) = 1 - e^{-\lambda x}$, $x > 0$, $\lambda > 0$ **(7x4)**

2. (a) Let X be a random variable that takes on 5 possible values with respective probabilities 0.35, 0.2, 0.2, 0.2, and 0.05. Also, let Y be a random variable that takes on 5 possible values with respective probabilities 0.05, 0.35, 0.1, 0.15, and 0.35. Find H(X) and H(Y) and verify whether H(Y) > H(X)

(b) In a binary communication system , a 0 or 1 is transmitted. Because of channel noise, a 0 can be received as a 1 and vice versa. Let m_1 and m_2 denote the events of transmitting 0 and 1, respectively. Let r_0 and r_1 denote the events of receiving 0 and 1, respectively. Let $P(m_0) = 0.5$, $P(r_1 | m_0) = 0.1$, and $P(r_0 | m_1) = 0.2$



(i) Find $P(r_0)$ and $P(r_1)$.

(ii) If a 0 was received, what is the probability that a 0 was sent ?

(iii) If a 1 was received, what is the probability that a 1 was sent ?

(iv) Calculate the probability of error P_e.

(v) Calculate the probability that the transmitted signal is correctly read at the receiver.

(9+9)

3. (a) Consider a random process X(t) defined by $X(t) = U\cos t + V\sin t, -\infty < t < \infty$ where U and V are independent r.v's, each of which assumes the values -2 and 1 with the probabilities 1/3 and 2/3 respectively. Show that X(t) is WSS but not strict-sense stationary.

(b) Suppose that the joint density of X and Y is given by :

$$f(x, y) = \frac{e^{-\frac{x}{y}} e^{-y}}{y}, \quad 0 < x < \infty, 0 < y < \infty \text{ compute } E[X | Y = y]$$

(9+9)

4. (a) Using infinite series method show that the Laplace transform

$$L\left\{\int_0^t \frac{\sin u}{u} du\right\} = \frac{1}{s}\tan^{-1}\frac{1}{s}$$

(b) Find the Fourier for the function defined by :

$f(x) = -x, \quad -\pi < x < 0$

$f(x) = 0, \quad 0 < x < \pi,$

$f(x + 2\pi) = f(x)$

(9+9)

5. (a) Consider the random process $X(t) = Y\cos \omega t,\ t \geq 0$ where ω is a constant and Y is a uniform random variable over (0, 1)

(i) Find E[X(t)]

(ii) Find the autocorrelation function R_x (t, s) of X(t).

(iii) Find the autocovariance function K(t, s) of X(t).

(b) Consider a Markov chain with state space {0, 1, 2} and transition probability

matrix $P = \begin{bmatrix} 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Show that state 0 is periodic with period 2. **(9+9)**

6. (a) Poisson arrivals with rate 28 per second enter a single queue for processing by one of two iid exponential servers, each with service rate 20 jobs per second. If a job arrives when the system is empty, it chooses one of the servers at random. The total number of jobs that the entire system can hold is only 3 (including any under service). Arrivals to a full system are lost. Evaluate the expected response time of a job that got admitted into the system

(b) Use the dual simplex method to solve the following linear programming problem

Maximize $Z = -2x_1 - 3x_2$

Subject to $x_1 + x_2 \geq 2$

$2x_1 + x_2 \leq 10$

$x_1 + x_2 \leq 8$

$x_1, x_2, x_3 \geq 0$ **(9+9)**

7. (a) Consider the following non-linear programming problem

Maximize $Z = \ln(x_1 + 1) + x_2$

Subject to $2x_1 + x_2 \leq 3$

$x_1, x_2 \geq 0$

Use Kuhn Tucker conditions and solve the problem.

(b) Consider the linear programming problem

Maximize $Z = 3x_1 + 13x_2$

Subject to $2x_1 + 9x_2 \leq 40$

$11x_1 - 8x_2 \leq 82$

$x_1, x_2 \geq 0$, x_1 and x_2 are integers

Solve by branch and bound method. **(9+9)**

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