

C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

a) Given that

$$A^T = \begin{bmatrix} 5 & 1 & 0 \\ 0 & -2 & -1 \\ 1 & 0 & 3 \end{bmatrix}, B^T = \begin{bmatrix} 6 & 3 & 1 \\ 1 & -15 & -5 \\ -2 & -1 & 10 \end{bmatrix}$$

Find the matrix multiplication of Matrix A with B. Also find the eigen value of matrix AB.

b) Find the domain and range of the function

$$f(x) = \frac{x}{1+x^2}.$$

c) Show that

$$\lim_{x \rightarrow 0} \frac{\log(1+x)}{x} = 1.$$

d) Find the equations of tangents to the curve $y = x^3 + 2x + 6$ which is perpendicular to the line $x + 14y + 4 = 0$.

e) Expand $\tan^{-1}x$ upto x^7 by maclaurine series.

f) Five salesman A, B, C, D and E of a company are considered for a three member trade delegation to represent the company in an international trade conference. Construct the sample space and find the probability that (i) A is selected (ii) Either A or B (not both) is selected.

g) The number of traffic accidents in Berkeley, California, in 10 randomly chosen non-rainy days in 1998 is as follows:

4, 0, 6, 5, 2, 1, 2, 0, 4, 3

Use these data to estimate the proportion of non-rainy days that had 2 or fewer accidents that year.

(7x4)

2.

a) Solve the system of equation equations

$$5x - 5y - 15z = 40$$

$$4x - 2y - 6z = 19$$

$$3x - 6y - 17z = 41$$

By Gauss – elimination method.

b) Find the rank of matrix A by Echolan method:

$$A = \begin{bmatrix} 2 & 1 & -7 & 14 \\ 6 & -3 & 19 & -38 \\ 1 & 0 & -3 & 6 \\ 2 & 1 & -6 & 12 \end{bmatrix}.$$

c) Find the inverse of a matrix A where

$$A = \begin{bmatrix} 5 & 7 & -6 \\ 3 & 1 & -2 \\ 1 & -5 & 5 \end{bmatrix}.$$

(5+7+6)

3.

a) Resolve

$$f(x) = \frac{2x - 1}{(x + 1)(x^2 + 2)}$$

into partial fractions.

b) If

$$y = b \tan^{-1} \left(\frac{x}{a} + \tan^{-1} \frac{y}{x} \right),$$

find $\frac{dy}{dx}$.

c) Using mean value theorem, prove that

$$\frac{b-a}{b} < \log \left(\frac{b}{a} \right) < \frac{b-a}{a},$$

Where $0 < a < b$.

(6+6+6)

4.

a) Find the equation of line which passes through the point (3, 4) and sum of its intercepts on the axes is 14.

b) Find the vertex, focus and direction of the parabola $4y^2 + 12x - 12y + 39 = 0$.

c) Find the equation of hyperbola, the length of whose latus rectum is 8 and eccentricity is $\frac{3}{\sqrt{5}}$.

(5+8+5)

5.

a) Test the convergence of the series

$$\sum_{n=1}^{\infty} \frac{1}{n^3} \left(\frac{n+2}{n+3} \right)^n.$$

b) Expand $\log x$ in powers of $(x-1)$ by Taylor's Theorem upto fourth degree terms.

c) Determine the Fourier series for the function

$$f(x) = \begin{cases} -1 + x, & -\pi < x < 0 \\ 1 + x, & 0 < x < \pi \end{cases}$$

Hence, deduce that $1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \dots = \frac{\pi}{4}$.

(5+5+8)

6.

a) Three newspapers R, G and O are published in a certain city. It is estimated from a survey that of the adult population: 20% read R, 16% read G, 14% read O, 8 % read R and G, 5% read R and O 4% read G and O, 2% read all three. What is probability that read at least one of newspapers?

b) What is Binomial distribution? Derive the first three moments about origin of binomial distribution. Also find the mean and variance.

c) Obtain the moment generating function of Normal distribution.

(5+7+6)

7.

a) The regression lines of y on x and x on y are respectively $y = a x + b$, $x = c y + d$. show that

$$\frac{\sigma_y}{\sigma_x} = \sqrt{a/c}, \bar{x} = \frac{bc+d}{1-ac} \text{ and } \bar{y} = \frac{ad+b}{1-ac}.$$

b) i) Suppose X_1, \dots, X_n are independent Poisson random variables each having mean λ . Determine the maximum likelihood estimator of λ .

ii) Let X_1, \dots, X_n be a random sample from a distribution with mean μ and variance σ^2 . Let \bar{X} be the sample mean, show that the expected value of the sample variance S^2 is equal to the population variance σ^2 .

c) A die is thrown 276 times and the results of these throws are given below:

| | | | | | | |
|-------------------------|----|----|----|----|----|----|
| No. appeared on the die | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | 40 | 32 | 29 | 59 | 57 | 59 |

Test whether the die is biased or not.

(5+7+6)