B3.2-R4: DISCRETE STRUCTURE

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) What is the Cartesian product *A* x *B* x *C*, where *A* is the set of all airlines and *B* and *C* are both the set of all cities in the United States? Give an example of how this Cartesian product can be used.

b) Consider the following collections of subsets of S = {1, 2, …, 8, 9}. Find which of the following is a partition of S?

i) [{1,3,5},{2,6},{4,8,9}]

ii) [{1,3,5},{2,4,6,8},{5,7,9}]

iii) [{1,3,5},{2,4,6,8},{7,9}]

c) Determine whether each of these functions is a bijection from R to R.

i) f(x)=2x + 1

ii) f(x)=x^2 + 1

iii) f(x)=x^3

iv) f(x)=(x^2 + 1)/(x^2 + 2)

d) Use K map to find the minimal sum for

$E_1 = x'yz + x'yz't + y'zt' + xyzt' + xy'z't'$

e) Suppose X={1,2,6,8,12} is ordered by divisibility and suppose Y={a,b,c,d,e} is isomorphic to X;say, the following function f is a similarity mapping from X onto Y:f={(1,e),(2,d),(6,b),(8,c),(12,a)}. Draw the Hasse diagram of Y.

f) For any words *u* and *v*, show that: (i) $|uv|=|u|+|v|$; (ii) $|uv|=|vu|$.

g) Let G be the directed graph with vertex set V(G)=(a,b,c,d,e,f,g) and edge set:

E(G)= {(a,a),(b,e),(a,e),(e,b),(g,c),(a,e),(d,f),(d,b),(g,g)}

- i) Identify any loops or parallel edges.
- ii) Are there any sources in G?
- iii) Are there any sinks in G?
- iv) Find the subgraph H of G determined by vertex set V={a,b,c,d}.

(7×4)

2.

a) Let a=8316 and b=10920.

- i) Find d=gcd(a,b), the greatest common divisor of a and b.
- ii) Find integers m and n such that d=ma + nb.
- iii) Find lcm(a,b), where lcm(a,b) is the least common multiple of a and b.

b) Find A(1,3) where A(mn,) for m=1 and n=3 in the Ackermann function defined as follows:

$$A(m,n) = \begin{cases} n+1 & if\ m=0 \\ A(m-1,1) & if\ m>0\ and\ n=0 \\ A(m-1,A(m,n-1)) & if\ m>0\ and\ n>0 \end{cases}$$

c) Suppose every student in a discrete maths class of 25 students is a freshman, a sophomore, or a junior.

i) Show that there are atleast 9 freshman, atleast 9 sophomores, or atleast 9 juniors in the class.

ii) Show that there are either at least 3 freshmen, atleast 19 sophomores, or atleast 5 juniors in the class

(6+6+6)

3.

a) i) Suppose A is the set of distinct letters in the word *elephant*, B is the set of distinct letters is the word *sycophant*, C is the set of distinct letters in the word *fantastic*, and D is the set of distinct letters in the word *student*. The universe U is the set of 26 lower-case letters of the English alphabet. Find

1) $A \cup B$
2) $A \cap C$
3) $A \cap (C \cup D)$
4) $(A \cup B \cup C \cup D)'$

ii) Find two finite set A and B such that $A \in B$ and $A \subset B$.

iii) Give a proof of or a counter example to the following statement:
$A \cap (B \cup C) = (A \cup B) \cap (A \cup C)$

b) Let p and q be the propositions, p: It is below freezing. q: It is snowing.
Write these propositions using p and q and logical connectives (including negations).

i) It is below freezing and snowing.
ii) It is below freezing but not snowing.
iii) It is not below freezing and it is not snowing.
iv) It is either snowing or below freezing (or both).
v) If it is below feezing, it is also snowing.
vi) Either it is below freezing or it is snowing, but it is not snowing if it is below freezing.
vii) That it is below freezing is necessary and sufficient for it to be snowing.

(9+9)

4.

a) Let $P(x)$,$Q(x)$, and $R(x)$ be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x)$,$Q(x)$, and $R(x)$, where the domain consists of all people.

i) No professors are ignorant.
ii) All ignorant people are vain.
iii) No professors are vain
iv) Does (c) follow from (a) and (b)?

b) Suppose that the number of bacteria in a colony triples every hour.

i) Set up a recurrence relation for the number of bacteria after n hours have elapsed.
ii) If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

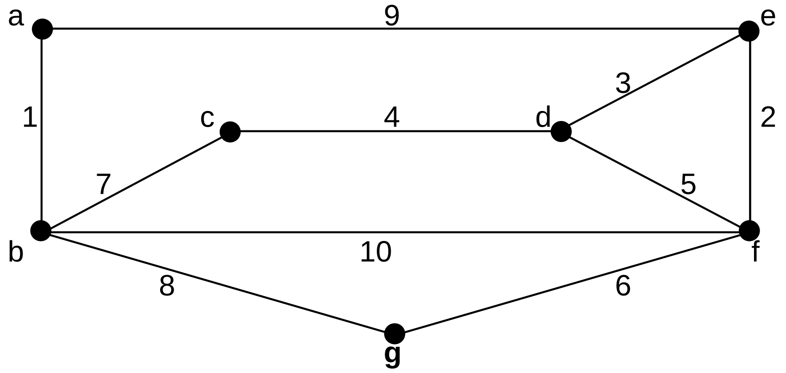
(9+9)

5.

a) How many bit strings of length 8 either start with 1 bit or end with the two bit 00?

b) Construct a table showing the interchanges that occur at each step when selection sort is applied to the following list: 5, 3, 4, 6, 2.

c) Use Kruskal's algorithm to find a minimal spanning tree for the following graph. What is the total weight of the minimal spanning tree?



(6+6+6)

6.

a) i) Draw a gate implementation for a One-Bit Equality Circuit: the output of this circuit is 1 if and only if both inputs are 0 or both inputs are 1.

ii) Find the canonical form for f=xy+ z'.

iii) Explicitly define the canonical form for f=xy+ z' by means of a truth table.

b) Consider the group G={1,2,3,4,5,6} under multiplication modulo 7.

i) Find the multiplication table of G.

ii) Find 2-1, 3-1, 6-1.

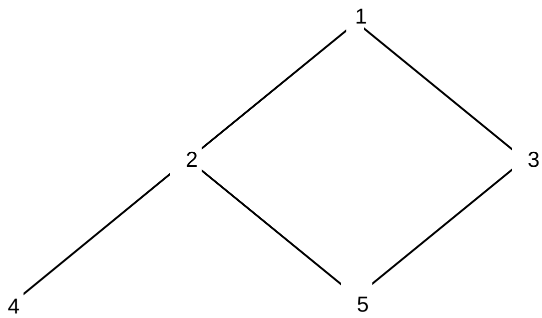
iii) Find the orders and subgroups generated by 2 and 3.

iv) Is G cyclic?

(9+9)

7.

a) Let A={1,2,3,4,5} be ordered by the Hasse diagram in following figure:



i) Insert the correct symbol, <, >, or || (not comparable), between pair of elements:

1) 1_____5; 2) 2_____3; 3) 4_____1; 4) 3_____4;

ii) Find all minimal and maximal elements of A.

iii) Does A have a first element or a last element?

iv) Let L(A) denote the collection of all linearly ordered subsets of A with 2 or more elements, and let L(A) be ordered by set inclusion. Draw the Hasse diagram of L(A).

b) Let M be the finite state machine with state table appearing in following figure:

F	*a*	*B*
S_0	S_1, x	S_2, y
S_1	S_3, y	S_1, z
S_2	S_1, z	S_0, x
S_3	S_0, z	S_2, x

i) Find the input set A, the state set S, the output set Z, and the initial state.

ii) Draw the state diagram D=D(M) of M.

iii) Suppose w=aababaabbab is an input word (string). Find the corresponding output word v.

(9+9)