B0-R4: BASIC MATHEMATICS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Express $\frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$ in the polar form $r(cos\theta + i\ sin\theta)$.

b) Evaluate $\lim_{x\to 0} \frac{xe^{x}-\log(1+x)}{x^2}$.

c) if a, b, c are all different. Then evaluate $\begin{vmatrix} a-b-c & 2b & 2c \\ 2a & b-c-a & 2c \\ 2a & 2b & c-a-b \end{vmatrix}$.

d) Using Cauchy integral test, test the convergence of the series $\Sigma \frac{n}{(n^2+1)^2}$.

e) Solve $x \sin x \frac{dy}{dx} + (x \cos x + \sin x)y = \sin x$.

f) Evaluate $\int_0^{\pi} \frac{x \sin x}{1+cos^2 x}\, dx$.

g) Determine a unit vector perpendicular to each of the vectors $2\hat{\imath} - \hat{\jmath} + \hat{k}$ and $3\hat{\imath} + 4\hat{\jmath} - \hat{k}$ and the sine of the angle between them.

(7x4)

2.

a) Find the values of μ and λ if the rank of A=$\begin{vmatrix} 1 & -2 & 3 & 1 \\ 2 & 1 & -1 & 2 \\ 6 & -2 & \lambda & \mu \end{vmatrix}$ is 2.

b) Use De moivre's theorem to solve the equations $x^4 - 1 = 0$.

c) Verify Langrange's Mean Value Theorem for the function $f(x) = (x-1)(x-2)(x-3)$ in (1, 4).

(6+6+6)

3.

a) Find the Eigen values and Eigen vectors of the matrix A=$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$.

b) Show that the matrix A=$\frac{1}{\sqrt{6}}\begin{vmatrix} \sqrt{2} & -1 & \sqrt{3} \\ \sqrt{2} & 2 & 0 \\ \sqrt{2} & -1 & -\sqrt{3} \end{vmatrix}$ is orthogonal.

(9+9)

4.

a) Find the maximum and minimum values of $\frac{x}{2}$ - $\sin x$ in $0 < x < 2\pi$.

b) Find the asymptotes of the curve y = $\frac{x^3}{x^2+x-2}$.

c) Evaluate $\int \frac{x^2 tan^{-1} x^3}{1+x^6} dx$.

(6+6+6)

5.

a) Solve the system of equations

$$\begin{aligned} x + y + z &= 7 \\ x + 2y + 3z &= 16 \\ x + 3y + 4z &= 20 \end{aligned}$$

by Cramer's rule.

b) Find the area of the smaller portion enclosed by the curves $y^2 = 8x$ and $x^2 + y^2 = 9$.

(9+9)

6.

a) Solve the differential equation $x^2 \frac{d^2y}{dx^2} + 3x\frac{dy}{dx} + y = (1-x)^2$.

b) Test the converges of the series

$$\frac{1}{\sqrt{1}+\sqrt{2}} + \frac{1}{\sqrt{2}+\sqrt{3}} + \frac{1}{\sqrt{3}+\sqrt{4}} + - - - - -.$$

c) Find the length of the loop of the curve $x = t^2$, $y = t - \frac{t^3}{3}$.

(6+6+6)

7.

a) Apply Maclaurin's theorem to show that

$$\tan(\left(\frac{\pi}{4} + x\right) = 1 + 2x + 2x^2 + \frac{8}{3}x^3 + \frac{10}{4}x^4 + - - - - - - -.$$

b) Find the equation of the hyperbola whose focus is (-1, 1), eccentricity=3 and the equation of the corresponding directrix is $x - y + 3 = 0$.

c) Find the vector equation of the line joining the points $\hat{\imath} - 2\hat{\jmath} + \hat{k}$ and $3\hat{k} + 2\hat{\jmath}$.

(6+6+6)