

C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

a) Find the rank of the following matrix:

$$A = \begin{bmatrix} 2 & 3 & 4 & -1 \\ 5 & 2 & 0 & -1 \\ -4 & 5 & 12 & -1 \end{bmatrix}$$

b) Test the convergence of the following infinite series:

$$\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}; \quad \forall x \neq 1.$$

c) Verify Cauchy's mean value theorem for the function e^x and e^{-x} in the interval (a, b) .

d) Evaluate $\lim_{x \rightarrow 0} \frac{xe^x - \log_e(1+x)}{x^2}$.

e) Find $\frac{dy}{dx}$ if $x^{2/3} + y^{2/3} = a^{2/3}$.

f) Six cards are drawn at random from a pack of 52 cards. What is the probability that three will be red and three will be black?

g) Find the equation of the parabola which is symmetric about the Y-axis and passes through the point $(2, -3)$.

(7x4)

2.

a) Test the consistency of the following system of linear equation and hence find the solution, if exists: $3x + 3y + 2z = 1$; $x + 2y = 4$; $10y + 3z = -2$; $2x - 3y - z = 5$.

b) Reduce the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$ to the diagonal form. Hence evaluate A^4 .

(9 + 9)

3.

a) Evaluate the integral: $\int_0^{\pi} \frac{x \sin^3 x}{1 + \cos^2 x} dx$.

b) Find the maximum and minimum values of $f(x) = x + \sin 2x$ on $[0, 2\pi]$.

c) Find asymptotes of the curve: $(x - y)^2(x + 2y - 1) = 3x + y - 7$.

(6+6+6)

4.

a) Discuss the convergence of the series: $x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \dots$

b) Expand $\log_e x$ in ascending powers of $(x-1)$ and hence evaluate $\log_e 1.1$ correct to four decimal places.

- c) Find the values of a and b such that the expansion of $\log_e(1+x) - \frac{x(1+ax)}{(1+bx)}$ in ascending powers of x begins with the term x^4 and hence find this term.

(8+5+5)

5.

- a) Find the equation of the circle which passes through the points (2, -2) and (3, 4) and whose centre lies on the line $x + y = 2$.
- b) If α and β are the roots of the quadratic equation $2x^2 - 5x + 7 = 0$, then find the quadratic equation whose roots are $2\alpha + 3\beta$ and $3\alpha + 2\beta$.
- c) Find the equation of the line perpendicular to the line $x - 2y + 3 = 0$ and passing through the point (1, -2).

(6+6+6)

6.

- a) The probability density function of a given variate X is
- | | | | | | | | |
|-------|-----|------|------|------|------|-------|-------|
| X: | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| P(X): | k | $3k$ | $5k$ | $7k$ | $9k$ | $11k$ | $13k$ |
- i) Find $P(X < 4)$; $P(X \geq 5)$; $P(3 < X \leq 6)$.
- ii) What will be the minimum value of k so that $P(X \leq 2) > 0.3$.
- b) The distribution of the number of road accidents per day in a city is Poisson distribution with mean 4. Find the number of days out of 100 days when there will be (i) no accidents (ii) at most 3 accidents (iii) at least 2 accidents (iv) between 2 and 5 accidents.
- c) Find the moment generating function of the discrete binomial distribution given by $P(x) = {}^n C_x p^x q^{n-x}$ (where $q = 1 - p$). Also find the first and second moments about the mean.

(5+8+5)

7.

- a) The following table gives age (X) in years of cars and annual maintenance cost (Y) in hundred rupees:
- | | | | | | |
|----|----|----|----|----|----|
| X: | 1 | 3 | 5 | 7 | 9 |
| Y: | 15 | 18 | 21 | 23 | 22 |
- Estimate the maintenance cost for a four year old car after finding the regression equation.
- b) The following table gives the number of accidents that took place in an industry during various days of week. Test if accidents are uniformly distributed over the week.
- | | | | | | | |
|------------------|--------|---------|-----------|----------|--------|----------|
| Days: | Monday | Tuesday | Wednesday | Thursday | Friday | Saturday |
| No. of Accidents | 14 | 18 | 12 | 11 | 15 | 14 |

(9+9)