B3.2-R4: DISCRETE STRUCTURE

«QP_SRLNO»

NOTE:

> 1. **Answer question 1 and any FOUR from questions 2 to 7.**
> 2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Using the set identities or Venn diagram show that (A – B) – C = A – (B U C).

b) Find the probability, that a family with five children does not have a boy, if the sexes of children are independent and if the probability of a boy is 0.51.

c) In how many ways can three disciplinary subjects be selected from the two groups of 6 subjects each, if at least one subject must be selected from each group?

d) Let f be a function from A={x | x is real and x >= -1} to B={x| x is real and x >= 0} given by f(a) = √(a+1). Find f^{-1}.

e) Find whether the following Propositional formula is a Tautology, Contradiction or else: You may use truth table.

$$\neg (p \wedge (q \vee p)) \leftrightarrow p$$

f) Let A = { x ∈ R | x ≠ 2 } and B = { x ∈ R| x ≠ 1}. Define f: A → B and g: B → A by f(x) = x/(x-2, g(x) = 2x/(x-1)

i) Find (f o g)(x)

ii) Are f and g inverses? Explain

g) Find a grammar that generates the language L= $\{a^n b^n \mid n \geq 1\}$.

(7x4)

2.

a) Prove by mathematical induction, that the Fibonacci number F_{3n} is divisible by 2.

b) Define the relation R on the set Z, by aRb if (a – b) is a nonnegative even integer. Verify that R defines a partial order. Is this partial order a total order?

c) Find the domain and range of the following function of a real variable:

g(x) = x |x|

(7+7+4)

3.

a) i) Define the asymptotic notation O (Big "oh"), and Ω (Omega) used to describe the asymptotic running time of algorithm.

ii) Prove the following identities:

A) $6n^2 / (\log n + 1) = O(n^2)$

B) log(n !) = O(n log(n))

b) Define ~ on Z by a ~ b if and only if 3a + b is a multiple of 4.

i) Prove that ~ defines an equivalence relation.

ii) Find the equivalence class of 2.

c) Show the steps of merge sort algorithm applied to the list: 2, 9, 1, 4, 6, 5, 3.

(7+7+4)

4.

a) Use mathematical induction to prove that for the recurrence relation $b_n = b_{n-1} + 2b_{n-2}$, $b_1=1$, $b_2= 3$, we have, $b_n < (5/2)^n$.

b) Let n be a positive integer and let D_n be the set of all positive divisors of n. Draw the Hasse diagrams for D_{30}. Determine whether the Hasse diagram represents a lattice giving reason?

c) Let A= {a, b, c} and let R and S be relations on A whose matrices are

$$M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \quad \text{and } M_S = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{pmatrix}$$

Find M_{SoR} and show that $M_{SoR} = M_R \odot M_S$

(7+7+4)

5.

a) In a gathering of 30 people, there are 104 different pairs of people who know each other. Show that some person must have at least seven acquaintances.

b) How many graphs are there on 4 nodes with degrees 1,1,2,2?

c) Prove that a planar graph on n nodes has at most 3n-6 edges.

(6+6+6)

6.

a) Twenty varieties of chocolates are available and Shagun wants to buy eight chocolates. How many choices does she have if his brother insists that at least one chocolate should have a cherry center?

b) Find the minimal sum-of – products forms for the Boolean function:

$$F(x,y,z,w)=xy'zw' + x'y'zw' + x'yzw' + x'yzw + xy'z'w'$$

c) Design a finite state machine having an output of 1 exactly when the input string received to that point ends in 101, assuming that the alphabet= { 0,1}.

(6+6+6)

7.

a) Define group. Show that every group need not be abelion.

b) What do you mean by order of an element of a group? Find the order of each element of the group G = {0, 1, 2, 3, 4, 5} with binary operation: addition module 6.

c) State Lagrange's theorem and use it to show any group of prime order have no proper subgroup.

(6+6+6)