C4-R4: ADVANCED ALGORITHMS

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Solve the recurrence relation using substitution method

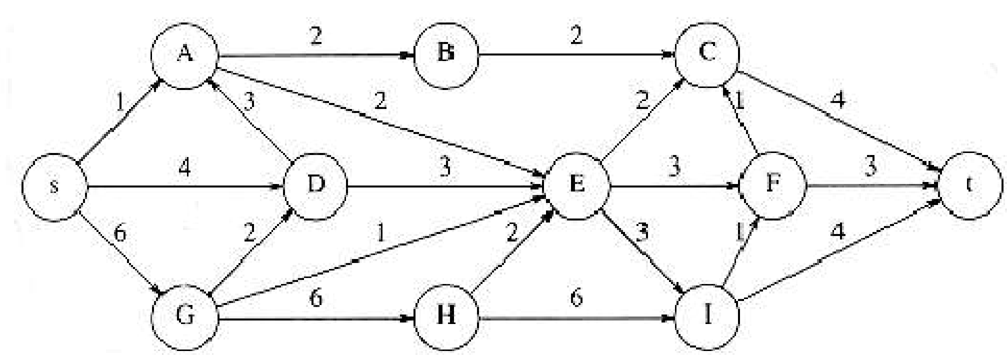
$$T(n) = \begin{cases} d & \text{if } n \le k \\ 7T(n/2) + cn^2 & \text{if } n > k \end{cases}.$$

b) Give an efficient algorithm to determine if there exists an integer i such that $a_i = i$ in an array of integers $a_1 < a_2 < a_3 < \ldots < a_n$. What is the running time of your algorithm?

c) A sorting algorithm is said to be stable if the original ordering for duplicate keys is preserved. Of the sorting algorithms Insertion Sort, Bubble Sort, Selection Sort, Shell Sort, Merge Sort, Quick Sort, Heap Sort, Bin Sort and Radix Sort, which of these are stable and which are not? For each one, describe either why it is or is not stable?

d) Show the result of the following sequence of instructions: union(1, 2), union(3, 4), union(3, 5), union(1, 7), union(3, 6), union(8, 9), union(1, 8), union(3, 10), union(3, 11), union(3, 12), union(3, 13), union(14, 15), union(16, 17), union(14, 16), union(1, 3), union(1, 14), when the unions are i). performed arbitrarily ii) performed by height and iii) performed by size.

e) Find a topological ordering for the graph given below:



f) Show the optimal binary search tree for the following words, where the frequency of occurrence is

Words	a	and	I	it	or
Frequency	0.18	0.19	0.23	0.21	0.19

g) Write an algorithm to determine whether a directed graph of |V| vertices contains a cycle. Your algorithm should run in O(|V| + |E|) time

(7x4)

2.

a) Show how heap-sort processes the input: 142, 543, 123, 65, 453, 879, 572, 434, 111, 242, 811, 102?

b) Is there any input for which heap-sort runs in O(nlogn) (in other words, are there any particularly good inputs for heap-sort)?

c) State the difference between internal sorting and external sorting.

(9+5+4)

3.

a) Prove that the amortized time bounds for Fibonacci heaps are O(1) for insert, merge, and decrease_key and O(log n) for delete_min.

b) Show how to merge two skew heaps with one top-down pass and reduce the merge cost to O(1) amortized time.

(9+9)

4.

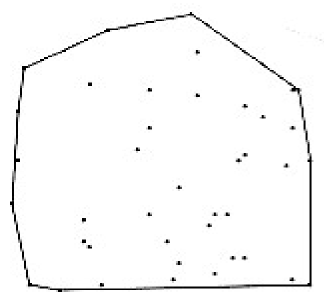
a) Suppose that the edges in an undirected graph G satisfy the triangle inequality: $c_{u,v} + c_{v,w} >= c_{u,w}$. Show how to compute a traveling salesman tour of cost at most twice optimal. (Hint: Construct a minimum spanning tree).

b) If P ≠ NP, then prove that for any constant p>1, there is no polynomial time approximation algorithm with approximation ratio p for the general traveling salesman problem.

(9+9)

5.

a) A convex polygon is a polygon with the property that any line segment whose endpoints are on the polygon lies entirely within the polygon. The convex hull problem consists of finding the smallest (area) convex polygon which encloses a set of points in the plane. The following figure shows the convex hull for a set of 40 points. Give an O(n log n) algorithm to find the convex hull.



b) Why is it important that Strassen's algorithm does not use commutativity in the multiplication of 2 X 2 matrices?

c) Show that if unions are performed by height, then the depth of any tree is O(log n).

(8+5+5)

6.

a) Let T be a tree (not necessarily distinct) in which a length is associated with each edge. Let S be a subset of the vertices of T and let T/S denote the forest that results when the vertices of S are deleted from T. Develop a greedy algorithm to find a minimal-cardinality subset S such that no forest in T/S has a root–to-leaf path whose length exceeds d.

b) Prove the correctness of algorithm designed in **Q 6. a)**.

c) A machine has n components. For each component, there are three suppliers. The weight of component I from supplier j is $W_{i,j}$, and its cost is $C_{i,j}$, 1 <= j <=3. The cost of machine is the sum of component costs, and its weight is the sum of the component weights. Assume that all its costs are positive integers. Write a dynamic programming algorithm to determine from which supplier to buy each component so as to have the lightest machine with cost no more than c. Assume that the costs are integer. What is the complexity of your algorithm? (6 + 3)

(6+3[6+3])

7. Write short notes on **any two** of the followings:

a) Backtracking problem

b) Boye-More Algorithm

c) Knapsack problem

(2x9)