C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A problem in a question paper is given to 3 students in a class to be solved. The probabilities of their solving the problem are 0.5, 0.7 and 0.8 respectively. Find the probability that the problem will be solved.

b) Suppose that X is a continuous random variable whose probability density function is given by

$$f(x) = \begin{cases} C(4x - 2x^2) & 0 < x < 2 \\ 0 & otherwise \end{cases}$$

i) What is the value of C?

ii) Find P{X > 1}.

c) If 10 fair dice are rolled, find the approximate probability that the sum obtained is between 30 and 40, inclusive.

d) At Standard, one of the largest restaurants in New Delhi, it normally takes 10 minutes to serve the order after receiving it. If the service time is exponentially distributed, what is the probability that the customer waiting time is (i) more than 8 minutes, (ii) 12 minutes or less, (iii) between 5 and 10 minutes?

e) Rewrite in standard form the following linear programming problem:

Minimize $z = 2x_1+x_2+4x_3$ subject to the constraints:

$-2x_1+4x_2 \leq 4$,

$x_1+2x_2+x_3 \geq 5$

$2x_1 +3x_3 \leq 2$.

x_1, $x_2 \geq 0$ and x_3 unrestricted in sign.

f) Find the Laplace transform of $f = \cos at$, $t \geq 0$.

g) Find the Fourier series expansion of the periodic function

$$f(x) = x, \quad -\pi \leq x \leq \pi, \; f(x + 2\pi) = f(x)$$

(7x4)

2.

a) The density function of X is given by

$$f(x) = \begin{cases} 1 & if\ 0 \leq x \leq 1 \\ 0 & otherwise \end{cases}$$

Find $E[e^X]$.

b) A system composed on n separate components is said to be parallel system if it functions when at least one of the components functions. For such a system, if component I, independent of other components, functions with probability pi, i=1, ……, n, what is the probability that the system functions?

c) For the following bivariate probability distribution of X and Y:

X \ Y	0	1	2	3
0	1/32	2/32	3/32	2/32
1	2/32	1/8	1/4	1/32
2	3/32	2/16	0	1/16

Find (i) $P(X \leq 1, Y = 3)$, (ii) $P(Y \leq 2)$, (iii) $P(X \leq 1 | Y \leq 2)$.

(6+6+6)

3.

a) Find the inverse Laplace transforms of the following functions using convolution

$$\frac{1}{(s^2 + w^2)^2}$$

b) Find the Fourier series expansion of the following periodic function of period 4

$$f(x) = \begin{cases} 2 + x & -2 \leq x \leq 0, \\ 2 - x & 0 < x \leq 2 \end{cases}$$

F(x+4) = f(x)

(9+9)

4.

a) Use simplex method to
Minimize $z = x_2 - 3x_3 + 2x_5$
subject to the constraints:

$3x_2 - x_3 + 2x_5 \leq 7,$
$-2x_2 + 4x_3 \leq 12,$
$-4x_2 + 3x_3 + 8x_5 \leq 10.$
$x_2, x_3, x_5 \geq 0.$

b) Problems arrive at a computing centre in Poisson fashion at an average rate of five per day. The rules of the computing centre are that any man waiting to get his problem solved must aid the man whose problem is being solved. If the time to solve a problem with one man has an exponential distribution with the mean time of 1/3 day, and if the average solving time is inversely proportional to the number of people working on the problem, approximate the expected time in the centre for a person entering the line.

(9+9)

5.

a) A straight line of length 4 units is given. Two points are taken at random on this line. Find the probability that the distance between them is greater than 3 units.

b) If two random variables have the joint density

$$f(x_1, x_2) = \begin{cases} x_1 x_2, & 0 < x < 1, 0 < x_2 < 2 \\ 0 & , \quad c.w \end{cases}$$

Find the probability that both random variables will take on values less than 1.

(9+9)

6.

a) Let X be a random variable that takes 3 possible values with respective probabilities 1/2,1/4 and 1/8. Find the entropy H(x) for the random variable X.

b) The amount of time, in hours, that a computer functions before breaking down is a continuous random variable with the probability density function given by

$$f(x) = \begin{cases} \lambda e^{-x/100} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

What is the probability that a computer will function between 50 and 150 hours before breaking down?

c) Optimize $z = 2x_1 + 3x_2 - (x_1^2 + x_2^2 + x_3^2)$ $subject\ to\ the\ constriants$:

$$x_1 + x_2 \leq 1,$$
$$2x_1 + 3x_2 \leq 6$$
$$2x_1 + 3x_2 \leq 6$$
$$x_1 \geq 0, x_2 \geq 0.$$

(4+5+9)

7.

a) A housewife buys three kinds of cereals: A, B and C. She never buys the same cereal on successive weeks. If she buys cereal A, then the next week she buys cereal B. However, if she buys either B or C, then the next week she is three times as likely to buy A as the other brand. Obtain the transition probability matrix and determine how she would buy each of the cereals in the long run.

b) Solve the following initial value problem using Laplace transform:

$y''-5y'+4y=e^{2t}$, $y(0)=19/12$, $y'(0)=8/3$.

(9+9)