

**C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES**

**NOTE:**

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

**Time: 3 Hours**

**Total Marks: 100**

**1.**

- a) Let the Eigen values of a 2x2 matrix  $A = \begin{pmatrix} a & b \\ 0 & c \end{pmatrix}$  be 1 and 2. What are the possible real values of  $a, b, c$ ?
- b) Evaluate  $\int_0^1 \frac{e^x}{1 + e^{2x}} dx$ .
- c) Find the derivative  $f'(0)$  for the function  $f(x) = x|x|$ .
- d) Four fair die are rolled simultaneously. What is the number of outcomes in which at least one of the dice shows 3?
- e) Examine the convergence of the following series  

$$\sum_{n=1}^{\infty} \frac{\cos^2 n}{n^2}$$
- f) Find  $\lim_{x \rightarrow 0} \frac{\sin(\sin(x))}{x}$ .
- g) What are the vertex and focus of the parabola  $y^2 = 4(x-3)$ .

**(7x4)**

**2.**

- a) Find the Eigen values and Eigen vectors of the following matrix:

$$\begin{bmatrix} 1 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 1 \end{bmatrix}$$

- b) Show that the points A, B, C with position vectors  $2\vec{i} - \vec{j} + \vec{k}$ ,  $\vec{i} - 3\vec{j} + 5\vec{k}$  and  $-\vec{i} - 4\vec{j} + 4\vec{k}$  respectively are the vertices of a right angled triangle.

**(12+6)**

**3.**

- a) For what real  $x$ , the following series is convergent

$$\sum_{n=0}^{\infty} \frac{(x-2)^n}{(n+1)4^{n+1}}$$

- b) For boys, the average number of absences in the first grade is 15 with a standard deviation of 7; for girls, the average number of absences is 10 with a standard deviation of 6. In a nationwide survey, suppose 100 boys and 50 girls are sampled. What is the probability that the male sample will have *at most* three more days of absences than the female sample, assuming that the population is normally distributed? It is given that the probability of a Z-score being -1.818 or less is about 0.035, where  $Z \sim N(0, 1)$ .

**(9+9)**

**4.**

- a) Sketch the region bounded by  $y = 2x - x^2$  and  $y = -3$ . Find the area of the region.
- b) A ladder 10 ft long rests against the vertical wall. If the base of the ladder slides away from the wall 1ft/sec then how fast the top of the ladder is sliding down the wall when the base is 6 ft from the wall (in ft/sec)?

**(9+9)**

5.

- a) Let a random variable  $X$  has PDF  $p(x) = \left(\frac{3}{4}\right) (1 - x^2)$ ,  $-1 \leq x \leq 1$ . Compute the mean, median, and variance of  $X$ .
- b) Let the equation of an ellipse be  $9x^2 + 16y^2 - 18x + 64y = 71$ . Find the center and lengths of major and minor axes.

(10+8)

6.

- a) Solve the following system of linear equations using Cramer's rule

$$x - 3y + z = 6$$

$$3x - y + z = 12$$

$$x + 5y - 3z = 6.$$

- b) Evaluate  $\lim_{x \rightarrow 0} \frac{x + \sin x}{\sqrt{x^2 + 2 \sin x + 1} - \sqrt{\sin^2 x - x + 1}}$ .

(9+9)

7.

- a) A binomial distribution with parameter  $n = 5$  satisfies the property  $8P(X = 4) = P(X = 2)$ . Find
- $p$
  - $P(x \geq 1)$ .
- b) A bin contains 8 light bulbs, 6 of which work properly but the other 2 are defective. You select to purchase 3 bulbs at random and without replacement from the bin. What is the probability that you get at least one defective bulb?

(10+8)