

B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.
3. Only Non-Programmable and Non-Storage type Scientific Calculator allowed.

Time: 3 Hours

Total Marks: 100

1.

- a) If $R = 6x^2y^2/z^4$ and errors in x, y, z are 0.03, 0.01, 0.02 respectively at $x = 3, y = 1, z = 2$. Calculate the absolute error and percentage error in evaluating R .
- b) A and B throw a fair coin alternatively. The first one who throws a head wins. If A starts, find his chances of winning.
- c) Obtain the value of $\sqrt{13}$ correct upto two decimal places using the Newton-Raphson method.
- d) Let X be a random variable with $E(X) = 10$ and $Var(X) = 25$. Find the positive values of a and b such that $Y = aX - b$ has expectation 0 and variance 1.
- e) If 8 fair coins are tossed, what is the probability of getting (i) exactly 3 heads, (ii) at least 2 heads?
- f) Find the second derivative, $f''(x)$, at $x=0.4$ for the function $f(x)$ given by

| | | | | | | |
|--------|-------|-------|-------|-------|-------|-------|
| x | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| $f(x)$ | 0.425 | 0.475 | 0.400 | 0.450 | 0.525 | 0.675 |

Using the approximation formula via Taylor Series.

- g) If the equations of the two lines of regression of Y on X and X on Y are $7x = 16y - 9$ and $5y - 4x = 3$ respectively. Find the coefficient of correlation.

(7x4)

2.

- a) Apply LU-factorization method to solve the equation $3x + 2y + 7z = 4, 2x + 3y + z = 5, 3x + 4y + z = 7$.
- b) Find the Lagrange interpolating polynomial of degree 2 approximating the function $y = \ln x$ for the following data. Also evaluate $y(2.7)$ and the error committed in this evaluation.

| | | | |
|-------------|---------|---------|---------|
| x | 2 | 2.5 | 3 |
| $y = \ln x$ | 0.69315 | 0.91629 | 1.09861 |

(9+9)

3.

- a) There are three boxes containing 1 white, 2 red, 3 black balls; 2 white, 3 red, 1 black balls; 3 white, 1 red, 2 black balls respectively. A box is chosen at random and from it, two balls are drawn at random. The two balls are 1 red and 1 white. What is the probability that they came from the second box?
- b) A curve is given by the following points (x, y) .

| | | | | | | | | | |
|-----|----|-----|----|-----|------|-----|----|-----|----|
| x | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| y | 23 | 19 | 14 | 11 | 12.5 | 16 | 19 | 20 | 20 |

Estimate the area bounded by the curve and the x -axis, respectively.

(9+9)

4. a) Solve the following system of equation by Gauss-Seidal method,
 $10x + 2y + z = 9$, $2x + 20y - 2z = -44$, $-2x + 3y + 10z = 22$.

- b) Let X be a random variable with the probability density function $f(x) = \begin{cases} \frac{1}{3}e^{-x/3}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$.

Find $P(X > 3)$, $E(X)$ and $Var(X)$.

(9+9)

5. a) Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of red balls drawn, find the joint probability distribution of (X, Y)

- b) The following data show the distribution of digits in numbers chosen at random from a telephone directory:

| | | | | | | | | | | |
|-----------|------|------|-----|-----|------|-----|------|-----|-----|-----|
| Digits | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| Frequency | 1026 | 1107 | 997 | 966 | 1075 | 933 | 1107 | 972 | 964 | 853 |

Test whether the digits may be taken to occur frequently and equally in the directory.

(9+9)

6. a) An electric firm manufactures light bulbs that have life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviation of 40 hours. Find the probability that a bulb burns (i) more than 834 hours (ii) between 778 and 834 hours.

- b) To find the strength of steel bars, 25 steel bars tested and it is found that the sample mean is 37.5 psi, and the standard deviation is 3.0 psi. Find the 95% confidence interval of the true mean yield strength of bars.

- c) Differentiate between the two types of statistical estimation of Parameters, Point Estimation and Interval Estimation. What are the point estimates of mean and variance, assuming the sample = $\{x_1, x_2, \dots, x_n\}$?

(8+5+5)

7. a) Find the constants a and b so that the following data is fitted to the curve $y = ae^{bx}$ by the method of least squares -

| | | | | |
|-----|--------|--------|---------|--------|
| x | 1 | 1.2 | 1.4 | 1.6 |
| y | 40.170 | 73.196 | 133.372 | 243.02 |

- b) Fit a Poisson distribution for the following data and test the goodness of fit at 5 % level of significance.

| | | | | | | |
|--------|-----|-----|-----|----|----|---|
| x | 0 | 1 | 2 | 3 | 4 | 5 |
| $f(x)$ | 110 | 170 | 130 | 60 | 23 | 7 |

(9+9)