B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Find the greatest lower bound and the least upper bound of the sets $\{3,\ 9,\ 12\}$ and $\{1,\ 2,\ 4,\ 5,\ 10\}$ if they exist in the poset $(Z_+, '/')$ where a/b means 'a divides b'.

b) Write the grammar that generates the set $\{a^n b^{2n} : n \geq 1\}$.

c) Let $R = \{(x,\ y) : x, y \in N \text{ and } x + y = 8\}$. Find the domain and range of *R*.

d) How many positive integers not exceeding 100 are divisible by 4 or 6?

e) Construct truth table to determine whether $(p \wedge q) \rightarrow p$ is a tautology.

f) A non-directed graph *G* has 8 edges. Find the number of vertices, if the degree of each vertex is 2.

g) Find the generating function for the sequence $\{3^0, 3^1,\ 3^2,\ 3^3\ ,............\}$.

(7x4)

2.

a) Show that if seven colors are used to paint 50 cars, at least eight cars will have the same color.

b) Let *N* be the set of all natural numbers and let *R* be a relation on $N \times N$ defined by $(a,\ b)\ R\ (c,\ d) \Leftrightarrow ad = bc$ for all $(a,\ b), (c,\ d) \in N \times N$. Show that R is an equivalence relation on $N \times N$.

c) Determine the validity of the following argument.

Either I will get good marks or I will not graduate. If I did not graduate I will go to Russia. I get good marks. Thus, I would not go to Russia.

(6+6+6)

3.

a) Prove that fourth root of unity namely $\{1,\ i,\ -1,\ -i\}$ is an abelian group under the set of complex number's.

b) Obtain the disjunctive normal form of the Boolean expression

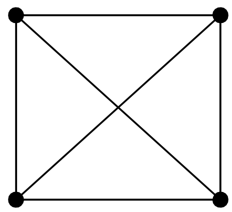
$$f(x,y,z) = \{(x' \vee y') \wedge z\} \vee \{x' \wedge (x \vee z)\}$$

c) What is the greatest common divisor of 119 and 272. Hence, determine their least common multiple.

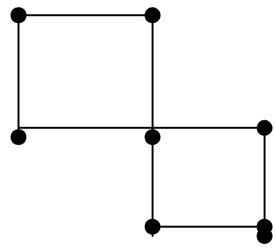
(6+6+6)

4.

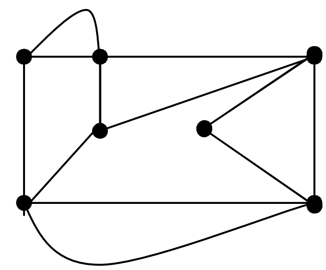
a) Which of the following graphs have an Euler circuit? Of these graphs which do not have Euler circuit, identify the graphs having Euler path. Give reasons.



(Fig. 1)



(Fig. 2)



(Fig. 3)

b) Use mathematical induction to prove that $7^{2n} + 2^{3n-3} \cdot 3^{n-1}$ is divisible by 25.

c) Solve the following recurrence relation $a_n = 5a_{n-1} - 6a_{n-2}$, where $a_0 = 3$ and $a_1 = 5$.

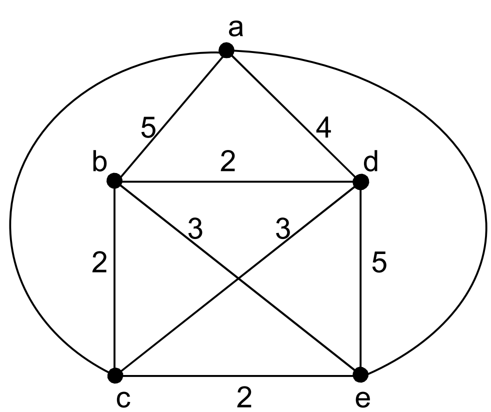
(6+6+6)

5.

a) For the finite state machine with transition table shown below, find all equivalent states and obtain an equivalent finite state machine with the smallest number of states-

State	Input		Output
	0	1	
A	F	B	0
B	D	C	0
C	G	B	0
D	E	A	1
E	D	A	0
F	A	G	1
G	C	H	1
H	A	H	1

b) Determine a minimum Hamiltonian circuit for the graph, if exists.



(9+9)

6.

a) Prove that for any $a,\ b$ in a Boolean Algebra, *B*

i) $a + a \cdot b = a$

ii) $a \cdot (a + b) = a$

b) Determine $a * b$ where $a_r = \begin{cases} 1\ , & 0 \le r \le 2 \\ 0\ , & r \ge 3 \end{cases}$ and $b_r = \begin{cases} 1\ , & 0 \le r \le 2 \\ 0\ , & r \ge 3 \end{cases}$

c) Find the order of each of the elements of group, G= $\{0,\ 1,\ 2,\ 3,\ 4,\ 5\}$ with the composition being addition modulo 6.

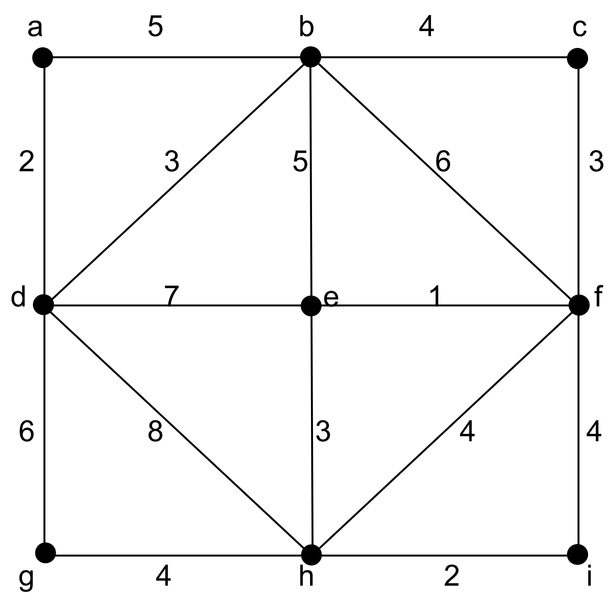
(6+6+6)

7.

a) Sort the following elements in ascending order using merge sort algorithm. Write the steps of the algorithm in detail.

8, 4, 1, 6, 9, 3, 2

b) Use Kruskal's algorithm to find a minimum spanning tree for the weighted graph given below:



(9+9)