NOTE:

> 1.   Answer question 1 and any FOUR from questions 2 to 7.
> 2.   Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours   **Total Marks: 100**

1.

a)   Evaluate $\lim\limits_{x \to 5} \left[\dfrac{\log x - \log 5}{x - 5} \right]$.

b)   Differentiate the following function with respect to x

$$e^x \log\left(1 + x^2\right).$$

c)   Solve the differential equation $\dfrac{dy}{dx} + 2x = e^{3x}$.

d)   Evaluate $\int \dfrac{1 - \sin 2x}{x + \cos^2 x} dx..$

e)   Find X if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2X + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$.

f)   Find a unit vector parallel to the vector $\left(\hat{i} - 2\hat{j}\right)$.

g)   If $z_1 = (1 - i)$ and $z_2 = (-2 + 4i)$ Find $\text{Im}\left(\dfrac{z_1 z_2}{\bar{z}_1} \right)$.

                                                                      (7x4)

2.

a)   Solve the following equations by Cramer's rule
$$2x + 3y = 10$$
$$x + 6y = 4$$

b)   Find the rank of the matrix by using elementary row operations
$$A = \begin{bmatrix} 2 & -2 & 0 & 6 \\ 4 & 2 & 0 & 2 \\ 1 & -1 & 0 & 3 \\ 1 & -2 & 1 & 2 \end{bmatrix}.$$

c)   Find the condition for the line $y = mx + c$ touches the parabola $y^2 = 4ax$.

                                                                     (4+8+6)

3.

a)   Find the radius and centre of the circle
$$z\bar{z} - (2 + 3i)z - (2 - 3i)\bar{z} + 9 = 0$$ where z is a complex variable.

b)   Find the point on the curve $y = 2x^2 - 6x - 4$ at which the tangent is parallel to x-axis.

c)   Find the value of *a* for which the function $f(x)=\begin{cases}\dfrac{1-\cos ax}{x\sin x} & \text{if } x \neq 0 \\[2mm] \dfrac{1}{2} & \text{if } x = 0\end{cases}$

is continuous at *x* = 0.

(6+6+6)

4.

a)   If $x\sqrt{1+y}+y\sqrt{1+x}=0$, prove that $\dfrac{dy}{dx}=-\dfrac{1}{(1+x)^2}$.

b)   Evaluate $\displaystyle\int_{0}^{\pi} \log(1+\cos x)dx$.

c)   Evaluate $\displaystyle\lim_{x \to 1}\left[\dfrac{1-x+\log x}{1-\sqrt{2x-x^2}}\right]$.

(6+6+6)

5.
a)   Find the volume and surface area of the solid generated by revolving the cycloid $x = a(t+\sin t), y = a(1+\cos t)$ about its base.

b)   Test the convergence of the series

$$\frac{1}{1^2+1}+\frac{\sqrt{2}}{2^2+1}+\frac{\sqrt{3}}{3^2+1}+\frac{\sqrt{4}}{4^2+1}+\cdots\cdots\cdots\cdots$$

(9+9)

6.
a)   Find the focus, directrices, eccentricity, latus rectum, and length of the axes of the hyperbola $25x^2-9y^2=225$.

b)   Solve the following differential equation

$$y-x\frac{dy}{dx}=x+y\frac{dy}{dx}.$$

(10+8)

7.
a)   The dot product of a vector with vectors $3i-5k$, $2i+7j$ and $i+j+k$ are respectively -1, 6 and 5. Find the vector.

b)   Find the limit when $n \to \infty$ of the series

$$\frac{n}{(n+1)^2}+\frac{n}{(n+2)^2}+\frac{n}{(n+3)^2}+\cdots\cdots\cdots\cdots\cdots+\frac{n}{(n+n)^2}.$$

c)   Discuss applicability of Rolle's Theorem for the function $f(x)=|x|$ in $[-1,1]$.

(6+6+6)