BE10-R4: APPLIED OPERATIONS RESEARCH

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) The annual demand for an item is 3200 units. The cost per unit of the item is Rs. 6/- and the inventory carrying charges is 25% per annum. If the cost of one order is Rs. 150/-, then determine:
   i) Economic order quantity
   ii) Number of orders per year

b) Determine the saddle point in following game:

$$\begin{array}{c} \\ \text{Player I} \end{array} \begin{array}{c} \text{Player II} \\ \begin{bmatrix} 3 & -1 & 4 & 6 & 7 \\ -1 & 8 & 2 & 4 & 12 \\ 16 & 8 & 6 & 14 & 12 \\ 1 & 11 & -4 & 2 & 1 \end{bmatrix} \end{array}$$

c) Use the graphical method to find an optimal solution of the following problem:

maximize $3x_1 + 5x_2$
subject to $6x_1 + 5x_2 \leq 30$
$3x_1 + 10x_2 \leq 30$
$x_1, x_2 \geq 0$

d) If in a particular single-server system, the arrival rate is, $\lambda = 5$ per hour and service rate is, $\mu = 8$ per hour. Assume the conditions for use of the single channel queuing model, find:
   i) the probability that the server is idle,
   ii) the Probability that there are at least two customers in the system.

e) Prepare network diagram from the information given in the following table:

Acticity (i – j)	Name of the Activity	Pre-requisites	Estimated time
1-2	A	None	3
1-3	B	None	5
1-4	C	None	4
2-5	D	A	2
3-5	E	B	3
4-6	F	C	9
5-7	G	D, E	8
3-6	H	B	7
6-7	I	H, F	9

f) Write the dual of the following linear programming problem:

maximize $Z = 3x_1 + 4x_2$
subject to $2x_1 + 3x_2 \leq 16$
$5x_1 + 2x_2 \geq 20$
$x_1, x_2 \geq 0.$

g) Perform only one complete iteration of steepest descent method to find the minimum value of f(x,y)= $2xy + 2x - x^2 - 2y^2$ starting at x= -1, y = 1.

(7x4)

2.

a) Using Big M method, solve the following linear programming problem:

minimize $Z = 4x_1 + x_2$

subject to $3x_1 + x_2 \geq 20$

$x_1 + 5x_2 \geq 15$

$x_1, x_2 \geq 0.$

b) A manufacturer has distribution centres located at Agra , Allahabad and Calcutta. These centres have available 8, 4 and 8 units of his products. His retails outlet requires the following number of units:
A,5; B,2; C,4; D,6; E,3. The shipping cost per unit(in Rupees) between each centre and outlet is given in the following table. Determine the optimal shipping cost:

Distribution Centres	**Retail outlets**				
	A	B	C	D	E
Agra	55	30	40	50	40
Allahabad	35	30	100	45	60
Calcutta	40	60	95	35	30

(9+9)

3.

a) A certain equipment needs five repair jobs which have to be assigned to five machines. The estimated time(in hours) that each mechanic requires to complete the repair job is given in the following table:

	J1	J2	J3	J4	J5
M1	7	5	9	8	11
M2	9	12	7	11	10
M3	8	5	4	6	9
M4	7	3	6	9	5
M5	4	6	7	5	11

Find the optimal assignment of Jobs on Machines so that total time is minimized. The machines cannot do partial jobs and a job cannot be done by more than one machine.

b) Telephone users arrive at a booth following a Poisson distribution with an average time of 5 minutes between one arrival and the next. The time taken for a telephone call is on an average 3 minutes and it follows an exponential distribution. What is the probability that the booth is busy? How many more booths should be established to reduce the waiting time to less than or equal to half of the present waiting time?

(9+9)

4.

a) The XYZ paints Ltd., would like to improve its inventory management policies for its supply of paint used for automobiles. Annual demand for such paint is 50,000 litres, and the paint, which costs Rs. 20 per litre, is used at a constant rate. Annual carrying costs are estimated at 15% of the value of paint held. Each order costs Rs. 80. Determine -

i) How much paint should be ordered each time?
ii) How often should paint be ordered?
iii) Time between two consecutive orders.
iv) What is the total annual cost associated with this policy?

b) A book binder has one printing press, one binding machine and manuscripts of 7 different books. The times required for performing printing and binding operations for different books are shown below:-

BOOK	PRINTING TIME (DAYS)	BINDING TIME (DAYS)
1	20	25
2	90	60
3	80	75
4	20	30
5	120	90
6	15	35
7	65	50

Decide the optimum sequence of processing of books in order to minimise the total time required to turn out all the books. Also compute the optimal time for all books to get printed and binded.

(8+10)

5.

a) A project has the following time schedule:

ACTIVITY	TIME (IN MONTHS)	ACTIVITY	TIME (IN MONTHS)
1-2	2	3-7	5
1-3	2	4-6	3
1-4	1	5-8	1
2-5	4	6-8	5
3-6	8	7-8	4

Construct PERT network and compute:

i) critical path and its duration

ii) total float for each activity.

b) A soft drink Company A calculates the market share of its two products against its major competitor Company B having three products to determine the impact of advertisement on its products against the other. The following is the pay off Company A over Company B.

		Company B		
		1	2	3
Company A	1	6	7	15
	2	20	12	10

What is the optimal strategy for the Company A as well as the competitor B in a competitive game situation?

(9+9)

6.

a) Use branch and bound method to solve the following integer program:

Minimize $Z = 7x_1 + 6x_2$

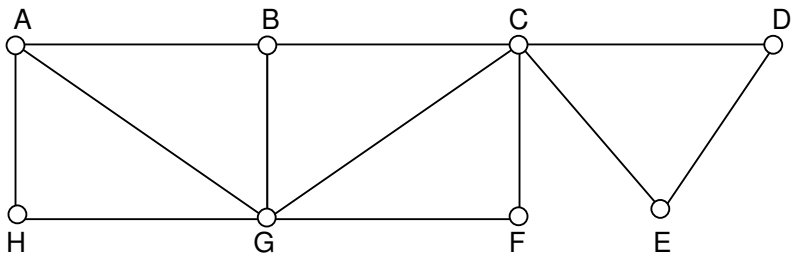
Subject to the constraints

$$2x_1 + 3x_2 \leq 12$$

$$6x_1 + 5x_2 \leq 30$$

where x_1, x_2 must be non-negative integers.

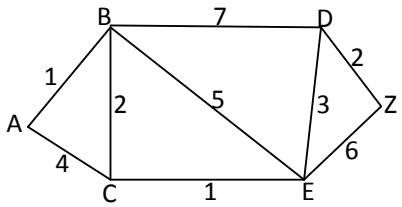
b) Find all sub graphs of the following graph obtained when one vertex is deleted at a time. Does G have any cut points? Explain.



(10+8)

7.

a) Find the shortest distance between source A and destination Z using Dijkstra's algorithm for following graph. Explain the steps in detail.



b) Customers arrive at a booking office window being manned by a single individual at a rate of 25 per hour. The time required to serve a customer has exponential distribution with a mean or 120 seconds. Find the average waiting time of a customer waiting for officer's service. Also find the probability that the officer remain idle. What is the expected number of customers on the officer counter?

(10+8)