NOTE:

> 1. Answer question 1 and any FOUR from questions 2 to 7.
> 2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours <div align="right">**Total Marks: 100**</div>

1.

a) Express the complex number in the form a+ib.
$$\frac{(1-i)(1+2i)}{(4+3i)}$$

b) Find $\lim_{x \to 0} \frac{\sqrt{1+x}-1-\left(\frac{x}{2}\right)}{x^2}$.

c) Let A = $\begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$. Find AA^T.

d) Test the convergence of the series $\frac{1}{1+x} + \frac{1}{2+x} + \frac{1}{3+x} + \cdots$

e) Solve the differential equation $x^2 \frac{dy}{dx} = 1 + y$.

f) Find the length of the arc of the parabola $y^2 = 4x$ from (0,0) to (1,2).

g) Using the properties of definite integrals, prove that $\int_{0}^{\pi/2} \frac{\sqrt{\tan x}}{1 + \sqrt{\tan x}} dx = \frac{\pi}{4}$.

<div align="right">**(7x4)**</div>

2.

a) Is the following matrix A invertible? If yes then find the inverse of A, where
$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$$

b) Evaluate the integral $\int \frac{dx}{(x^2 + 1)(x - 2)}$.

<div align="right">**(8+10)**</div>

3.

a) Determine the points of minima, maxima, and inflection point, if exist, for the function
$f(x) = x^4 - 4x^3 + 10, x \in R$.

b) Determine the asymptotes, if any, of the curve
$y(x - y)^2 = x + y$

<div align="right">**(9+9)**</div>

4.

a) Find the area of the region enclosed by the parabola $y = 2 - x^2$ and the line $y = -x$.

b) Expand f(x) $\tan x$ in the powers of $\left(x - \frac{\pi}{4}\right)$ upto first three terms using Taylor's series about the point $x = \frac{\pi}{4}$.

<div align="right">**(9+9)**</div>

5.

a) Solve the differential equation $\sqrt{1+x^2} + xy\dfrac{dy}{dx} = 0$.

b) Show that the following function is not differentiable at x=0.

$$f(x) = \dfrac{x(e^{\frac{1}{x}}-1)}{(e^{\frac{1}{x}}+1)} \text{ , } x\neq 0, \ f(0)=0.$$

Is this function continuous at x=0? Discuss.

(10+8)

6.

a) Find the projection of the vector $6\hat{i}+3\hat{j}+2\hat{k}$ onto the vector $\hat{i}-2\hat{j}-2\hat{k}$.

b) Find the equation of the parabola whose vertex is (0,2) and focus is (0,6).

c) Find the coordinates of the point where the line $x=\dfrac{8}{3}+2t$, y= - 2t , z = 1+t Intersects the plane

$$3x+2y+6z = 6.$$

(6+6+6)

7.

a) Find the eigenvalues of the matrix

$$\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{bmatrix}$$

b) Find the equation for the hyperbola centered at origin, with one focus at (3,0) and the line x=1 as the corresponding directrix.

(9+9)