

CE1.1-R4: DIGITAL SIGNAL PROCESSING

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) Derive the relationship between Discrete Fourier Transform (DFT) and z-transform.
- b) For the analog transfer function

$$H(s) = \frac{1}{(s+1)(s+2)}$$

- c) Determine system function, $H(z)$ using impulse invariant technique. Assume $T = 1$ second.
- Obtain a cascade realization of the system characterized by the transfer function

$$H(z) = \frac{2(z+2)}{z(z-0.1)(z+0.5)(z+0.4)}$$

- d) Determine the particular solution of the first order difference equation

$$y(n) + a_1 y(n-1) = x(n), \quad |a_1| < 1$$

- e) When the input $x(n)$ is unit step sequence: $x(n) = u(n)$.
Determine and sketch the convolution $y(n)$ of the signals, graphically.

$$x(n) = \begin{cases} \frac{1}{3}n, & 0 \leq n \leq 6 \\ 0, & \text{elsewhere} \end{cases}$$

$$h(n) = \begin{cases} 1, & -2 \leq n \leq 2 \\ 0, & \text{elsewhere} \end{cases}$$

- f) Sketch the pole-zero plot using z-transform. For the signal $x(n)$, where $a > 0$.

$$x(n) = \begin{cases} a^n, & 0 \leq n \leq M-1 \\ 0, & \text{elsewhere} \end{cases}$$

- g) Determine inverse z-transform of $X(z)$ using residue method, where $x(n)$ is causal sequence.

$$X(z) = \frac{z}{(z-1)(z-2)}$$

(7x4)

2.

- a) Design direct form-I and direct form- II realization of the second order filter given by

$$y(n) = 2b \cos w_0 y(n-1) - b^2 y(n-2) + x(n) - b \cos w_0 x(n-1)$$

- b) Determine the Fourier transform for the double exponential pulse whose function is given by

$$f(t) = e^{-a|t|}$$

- c) Draw magnitude and phase response of $F(w)$.
For the given system: $T[x(n)] = x(n)u(n)$, verify the following characteristics:
 - i) stable
 - ii) causal
 - iii) linear
 - iv) time invariant
 - v) memory less

(6+6+6)

3.

- a) Consider a finite duration sequence $x(n)$ of length P such that $x(n) = 0$ for $n < 0$ and $n \geq P$. To compute samples of the Fourier Transform at the N equally spaced frequencies

$$w_k = \frac{2\pi k}{N}, \quad k = 0, 1, \dots, N-1$$

Determine and justify one procedure for computing the N samples of the Fourier Transform of the following two cases:

- i) $N > P$
ii) $N < P$
- b) Determine the z -transform and its Region Of Convergence (ROC) of the following sequence:

i) $x(n) = 2^n u(-n)$

ii) $u(n+10) - u(n+5)$

- c) Design a digital Butterworth filter that satisfies the following constraint using bilinear transformation. Assume $T = 1$ second.

$$0.9 \leq |H(e^{jw})| \leq 1 \quad 0 \leq w \leq \pi/2$$

$$|H(e^{jw})| \leq 0.2 \quad 3\pi/4 \leq w \leq \pi$$

(6+6+6)

4.

- a) Write short note on "Contribution of Digital signal processing in Biomedical applications".
b) Derive single stage lattice filter and draw generalized as an extension form of lattice FIR filter. Determine lattice coefficients into direct-form coefficients.
c) Obtain the cascade and parallel realizations of the given system

$$H(z) = \frac{1 + \frac{1}{4}z^{-1}}{\left(1 + \frac{1}{2}z^{-1}\right)\left(1 + \frac{1}{2}z^{-1} + \frac{1}{4}z^{-2}\right)}$$

(4+7+7)

5.

- a) Compute recursively the zero state response of the system described by the difference equation

$$y(n) + \frac{1}{2}y(n-1) = x(n) + 2x(n-2)$$

To the input, $x(n)$ given as

$$x(n) = \{1, 2, 3, 4, 2, 1\}$$

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- b) Design a flow graph or butterfly diagram for a decomposing the decimation-in-frequency Fast Fourier Transform (FFT) algorithm for $N = 6$.

i) $N = 3 \times 2$

ii) $N = 2 \times 3$

(8+10)

6.

- a) Design a high pass digital FIR filter using Kaiser Window satisfying the specifications given below:

Passband cut-off frequency, $f_p = 3200$ Hz

Stopband cut-off frequency, $f_s = 1600$ Hz

Passband ripple, $A_p = 0.1$ dB

Stopband attenuation, $A_s = 40$ dB and

Sampling frequency, $F = 10000$ Hz.

- b) Describe advantages and disadvantages of orthogonal frequency division multiplexing (OFDM) as multicarrier modulation technique used in wireless communication systems.

- c) Determine the Fourier Transform of the signal

$$x(n) = \begin{cases} A, & -M \leq n \leq M \\ 0, & elsewhere \end{cases}$$

(7+7+4)

7.

- a) What is wavelet? Explain significance of Digital Wavelet Transform (DWT) in multi resolution analysis.

- b) Write short note on applications of Multirate signal processing.

- c) Determine the linear and circular convolution of the sequences $x_1(n)$ and $x_2(n)$ using matrix multiplication method.

$$x_1(n) = \{\underset{\uparrow}{1}, 2, 4\} \text{ and } x_2(n) = \{\underset{\uparrow}{1}, 2\}.$$

(7+4+7)