C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A box contains 'x' good VLSI chips and 'y' defective chips. If 'z' chips are selected at random, with $z < x$, then find the probability that no selected chip is defective.

b) In answering a question on a multiple choice test a student either knows the answer or guesses. Let p be the probability that she knows the answer and $(1- p)$ the probability that she guesses. Assume that a student who guesses at the answer will be correct with probability $1/m$, where m is the number of multiple-choice alternatives. Use the Baye's theorem to find the conditional probability that a student knew the answer to a question given that she has answered it correctly?

c) Weather is considered to be a two state Markov chain. Suppose that if it rains today then will rain tomorrow with probability α. And if it does not rain today then it will rain tomorrow with probability β. If we say that the process is in state 0 when it rains and state 1 when it does not rain then the preceding is a two-state Markov chain whose transition matrix P is given by

$$\begin{pmatrix} \alpha & 1-\alpha \\ \beta & 1-\beta \end{pmatrix}$$

If α=0.7 and β=0.4 then calculate the probability that it will rain for three days from today given that it is raining today.

d) Suppose that people immigrate into a territory at a Poisson rate λ=1 per day. (i) what is the expected time until the 10^{th} immigrant arrives? (ii) What is the probability that the elapsed time between the 10^{th} and 11^{th} arrival exceeds two days?

e) Convert the following linear programming problem into the *standard form*:

$$\begin{aligned} \text{maximize} \quad & 2x_1 + x_2 \\ \text{subject to} \quad & x_1 \le 2 \\ & x_1 + x_2 \le 3 \\ & x_1 + 2x_2 \le 5 \\ & x_1, x_2 \ge 0 \end{aligned}$$

f) Let $f(t) = (\frac{1}{2})(e^{at} + e^{-at})$. Find the Laplace transform of f.

g) Find E[X], where X is the outcome when a fair die with six faces is rolled once.

(7x4)

2.

a) The capacity of a wireless communication channel is 20000 bits per second (bps). This channel is used to transmit 8-bit characters. The application calls for traffic from many devices to be sent on the channel with a total volume of 120000 characters per minute. Calculate the average number of characters waiting in the channel and the average transmission time per character (including waiting time).

b) Let X denote a random variable that takes on any of the values -1, 0, 1 with respective probabilities
$P\{X= -1\} = 0.2$, $P\{X=0\}=0.5$, $P\{X=1\}=0.3$. Compute $E[X^2]$.

c) If 3 married couples are arranged in a straight line, then find the probability that no husband site next to his wife.

(6+6+6)

3.

a) Find the Inverse Laplace Transform of $F(s) = \frac{s+7}{s^2 - 3s - 10}$. Show the steps involved in it.

b) Find the Fourier coefficients of the periodic function $f(x)$

$f(x) = -1$ if $-\pi < x < 0$

$f(x) = 1$ if $0 < x < \pi$

and $f(x) = f(x+2\pi)$,

Also find the Fourier series of f.

(9+9)

4.

a) Consider the following linear program

maximize $2x_1 + 5x_2$

subject to $x_1 \le 4$

$x_2 \le 6$

$x_1 + x_2 \le 8$

$x_1 + x_2 \ge 1$

$x_1 , x_2 \ge 0$

Solve it using the simplex method.

b) An investor has 20 thousand dollars to invest among 4 possible investments. Each investment must be in units of thousand dollars. If the total 20 thousand is to be invested, how many different investment strategies are possible? Give reasons

(10+8)

5.

a) Let X be a gamma random variable with parameters α and λ. Calculate E[X] and Var[X].

b) The joint density of X and Y, two continuous random variables is given by

$f(x,y) = e^{-(x+y)}$ for $0 < x < \infty$ and $0 < y < \infty$

$f(x,y) = 0$ otherwise

Determine the dentisty function for the random variable $\frac{X}{Y}$.

(9+9)

6.

a) Let X be a random variable that takes on 3 possible values with respective probabilities 0.35, 0.75, and 0.5. Find the entropy H(X) for the random variable X.

b) Machines in a factory break down at an exponential rate of 6 per hour. There is only one repairman. He fixes the machine at an exponential rate of 8 per hour. The cost incurred in production loss when machine was out of service is Rs. 1000 per hour per machine. What is the average cost rate due to the failed machines?

c) Write the Kuhn-Tucker conditions for the following minimization problem:

minimize $f(x) = x_1^2 + x_2^2$

subject to $g_1(x) = x_1 + x_2 - 2 \le 0$

$g_2(x) = 1 - x_1 \le 0$

$g_3(x) = 2 - x_2 \le 0$

(6+6+6)

7.

a) Use the method of Laplace transform to solve the following initial value problem:

$$y''+3y'+2y=e^{-t},\ y(0)=-1,\ y'(0)=1.$$

b) Assume that a computer system is in one of three states: busy, idle, or undergoing repair, respectively denoted by states 0, 1, and 2. Observing its state at a fixed time every day, we believe that the system approximately behaves like a homogeneous Markov chain with the transition probability matrix:

$$P = \begin{pmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0.0 & 0.4 \end{pmatrix}$$

Prove that the chain is irreducible, and determine the steady-state probabilities.

(9+9)