C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) In a college entrance examination each candidate is admitted or rejected according to whether he has passed or failed the test. Of the candidates who are really capable, 80% passed the test and of the incapable, 25% pass the test. Given the 40% of the candidates are really capable, find the proportion of capable college students, using Baye's theorem.

b) The joint probability function of two discrete random variables X and Y is given by

$$f(x, y) = C(2x+y),\ 0 \le x \le 2,\ 0 \le y \le 3$$
$$= 0, \text{ otherwise}$$

Find the value of C.

c) Find the Laplace transform of the function

$$f(t) = 6e^{-5t} + 5t^3 - 9$$

d) A Stochastic process is described by

$$X(t) = A \sin t + B \cos t$$

where A and B are independent random variables with zero means and equal standard deviations. Show that the process is stationary of the second degree.

e) Write the dual of the following primal problem

Minimize $Z = 2x_1 + 6x_2$

Subject to $9x_1 + 3x_2 \ge 20$

$2x_1 + 7x_2 = 40$

$x_1 \ge 0,\ x_2 \ge 0$

f) The arrival rate of customers at the single window booking counter of a two wheeler agency follows Poison distribution and the service time follows exponential distribution and hence, the service rate also follows Poisson distribution. The arrival rate and the service rate are 25 customers per hour and 35 customers per hours, respectively. Find the following:

i) Average number of waiting customers in the queue.

ii) Average waiting time per customers in the queue.

g) A random process gives measurements x between 0 and 1 with a probability density function

$$f(x) = 10x - 4,\ 0 \le x \le 1$$

Find a number k, $0 < k < 1$, such that $P[X \le k] = 1/2$.

(7x4)

2.

a) Customers arrive at the First Class Ticket Counter of a Theatre at the rate of 12 per hour. There is one clerk serving the customers at the rate of 30 per hour.

i) What is the probability that there is no customer in the counter?

ii) What is the probability that there are more than 2 customers in the counter?

iii) What is the probability that there is no customer waiting to be served?

b) Use simplex method to solve the following Linear Programming Problem:

Maximize $z = 4x_1 + 10x_2$

Subject to the constraints:

$2x_1 + x_2 \le 50$

$2x_1 + 5x_2 \le 100$

$2x_1 + 3x_2 \le 90$

$x_1, x_2 \ge 0.$

(9+9)

3.

a) The occurrence of rain in a city on a day is dependent upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is

Event: (Rain)	*No Rain*	*1 cm.*	*2 cm.*	*3 cm.*	*4 cm.*	*5 cm.*
Probability	0.50	0.25	0.15	0.05	0.03	0.02

If it did not rain the previous day, the rain distribution is:

Event:(Rain)	*No Rain*	*1 cm.*	*2 cm.*	*3 cm.*
Probability:	0.75	0.15	0.06	0.04

Simulate the city's weather for 10 days and determine by simulation the total days without rain as well as the total rainfall during the period. Use the following random numbers for simulation:

67 63 39 55 29 78 70 06 78 76

Assume that for the first day of the simulation it had not rained the day before.

b) A transmitter has an alphabet consisting of three letters $[x_1, x_2, x_3]$ and the receiver has an alphabet of two letters $[y_1, y_2]$. The joint probabilities for the communication are given below:

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \end{array} \begin{array}{c} \begin{array}{cc} y_1 & \;\; y_2 \end{array} \\ \begin{bmatrix} 0.25 & 0.00 \\ 0.10 & 0.30 \\ 0.00 & 0.05 \end{bmatrix} \end{array}$$

Determine the different entropies for this channel (Assume that 0 log 0 = 0).

(9+9)

4.

a) Use branch and bound method to solve the following linear integer programming problem:

Maximize $z = 7x_1 + 9x_2$

subject to the constraints:

$-x_1 + 3x_2 \le 6$

$7x_1 + x_2 \le 35$

$x_2 \le 7$

$x_1, x_2 \ge 0$ and are integers.

b) For a > 0, verify that the following function is a Probability distribution function.

$$F(x) = \begin{bmatrix} 0 & , & x < a \\ 1/2[x/a + 1], & & -a \le x \le a\,. \\ 1 & , & x > a \end{bmatrix}$$

(10+8)

5.

a) The process X(t) whose probability distribution under certain condition is given by

$$P[X(t)=n] = \frac{(at)^{n-1}}{(1+at)^{n+1}}, n = 1,2,...$$

$$= \frac{at}{1+at}, n = 0$$

Show that it is not stationary.

b) The school of International Studies for Population found out by its survey that the mobility of the population of a state to the village, town and city is in the following percentages:

From	To		
	Village	Town	City
Village	50%	30%	20%
Town	10%	70%	20%
City	10%	40%	50%

What will be the proportion of population in village, town and city after two years, given that the present population has proportions of 0.7, 0.2 and 0.1 in the village, town and city respectively? What will be the respective proportions in the long run?

(9+9)

6.

a) Find the inverse Laplace transform of

$$\frac{30}{s^7} + \frac{8}{s - 4}$$

b) Find a Fourier series for the function

$F(x) = x - \pi$ when $-\pi < x < 0$,

$= \pi - x$ when $0 < x < \pi$.

(6+12)

7.

a) Determine x_1, x_2 and x_3 so as to

Maximize $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

subject to the constraints:

$x_1 + x_2 \le 2$

$x_1 + 3x_2 \le 12$

$x_1, x_2 \ge 0$.

b) Babies are born in a sparsely populated state at the rate of one birth every 12 minutes. The time between births follows an exponential distribution. Find the following:

i) The average number of births in a year.

ii) The probability that no births will occur in a year.

iii) The probability of issuing 50 birth certificates in 3 hours given that 40 certificates were issued during the first 2 hours of the 3 hour period.

(9+9)