

C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

a) If $A = \begin{bmatrix} 1 & 2 \\ -2 & 3 \end{bmatrix}$, $B = \begin{bmatrix} 2 & 1 \\ 2 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} -3 & 1 \\ 2 & 3 \end{bmatrix}$, then verify that $A(B+C) = AB+AC$.

b) Show that the series $\sum_{n=1}^{\infty} \sqrt{\frac{n}{13(n^2+1)}}$ is divergent.

c) Solve the following system of linear equations by using Cramer's Rule:

System is

$$3x + y + 2z = 3,$$

$$2x - 3y - z = -3,$$

$$x + 2y + z = 4$$

d) A sample of 900 children is found to have a mean height 3.4 m. Can it be reasonably a truly random sample from a large children population with mean height 3.25 m and standard deviation 1.61 m at 5% level of significance? Give reason.

e) Evaluate $\lim_{x \rightarrow \infty} (1+x)^{\frac{1}{x}}$.

f) Show that the function defined by $f(x) = |x| + |x-1|$ is continuous at $x=1$.

g) A bag contains 10 white and 15 black balls. Two balls are drawn in succession. What is the probability that the first ball is white and the second ball is black?

(7x4)

2.

a) The centre of a circle is (4, 6) and the circle passes through the centre of another circle $x^2 + y^2 - 4x - 6y - 10 = 0$. Find the equation of the circle.

b) The equation $x^2 + 9y^2 - 4x - 72y + 139 = 0$ represents an ellipse. Find the lengths of its major and minor axis.

c) Find an equation of the parabola whose vertex is (0, 2) and focus is (0, 6).

(6+6+6)

3.

- a) Express the matrix $\begin{bmatrix} 1 & 4 & 5 \\ 2 & 0 & 3 \\ 5 & 7 & 2 \end{bmatrix}$ as the sum of a symmetric and a skew-symmetric matrix.

Explain the steps clearly.

- b) Test the convergence of series

i) $\sum_{n=0}^{\infty} ne^{-n^2}$

ii) $\sum_{n=1}^{\infty} \frac{n^2}{3^n}$

- c) Verify the Cayley-Hamilton theorem for a matrix $\begin{bmatrix} 5 & 6 \\ 4 & 9 \end{bmatrix}$.

(6+6+6)

4.

- a) Define the value of $f(2)$ such that the function $f(x) = \frac{x^3 - 8}{x - 2}$ is continuous at the point $x=2$.

- b) Evaluate the integral $\int_0^{\pi/2} \log(\tan x) dx$..

- c) The probability that machine A will be performing an usual function in 5 years time is $\frac{1}{4}$, while the probability that machine B will still be operating usefully at the end of the same period is $\frac{1}{3}$. Find the probability in the following cases that in 5 years time:

- i) Both machines will be performing an usual function.
ii) Neither will be operating.

(4+7+7)

5.

- a) Find the sum of the series $\sum_{n=0}^{\infty} \left(\frac{1}{2^n} + \frac{1}{3^n} \right)$.

- b) Verify the Lagrange's Mean value theorem for the function $f(x) = lx^2 + mx + n$, $x \in [a, b]$.

- c) Expand $f(x) = x^4 + 3x^3 - 12x^2 - 9x + 10$ in powers of $(x-5)$.

(6+6+6)

6.

a) Calculate the mean and standard deviation of the following frequency distribution:

Weekly wages in Rs.	No. of men
4.5-12.5	4
12.5-20.5	24
20.5-28.5	21
28.5-36.5	18
36.5-44.5	5
44.5-52.5	3
52.5-60.5	5
60.5-68.5	8
68.5-76.5	2

b) An insurance company found that only 0.01% of the population is involved in a certain type of accident each year. If its 1000 policy holders were randomly selected from the population, what is the probability that not more than two of its clients are involved in such an accident next year? (given that $e^{-0.1} = 0.9048$).

(10+8)

7.

a) In a normal distribution, 31% of the items have values under 45 and 8% items have values over 64. Find the mean and standard deviation of the distribution.

b) In a study between the amount of rain fall and the quantity of air pollution removed the following data were collected.

Daily rainfall in 0.01 cm	4.3	4.5	5.9	5.6	6.1	5.2
Pollution removed (mg/m^3)	10	11	12	11	13	11

Find the regression line of y on x.

(9+9)