CE1.1-R4: DIGITAL SIGNAL PROCESSING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Determine y[n] for n ≥ 0 when x[n] = δ[n] and y[n] = 0 for n < 0, for the given linear constant-coefficient difference equation:

$$y[n]-\frac{3}{4}y[n-1]+\frac{1}{8}y[n-2]=2x[n-1]$$

b) Sketch and label carefully each of the following signals:

i) $x[n]u[2-n]$

ii) $x[n-1]\delta[n-3]$

For given, $x[n]=\{...0,1,\underset{\uparrow}{1},1,1,1,0.5,0..\}$

c) Determine linear convolution of following two sequences using z-transform.

$$x_1[n]=a^n u[n] \ and \ x_2[n]=u[n]$$

Sketch the pole zero plot of the resultant signal.

d) Draw transposed form for given first order system:

$$H(z)=\frac{1}{1-a z^{-1}}$$

e) What is the importance of "zero padding" in the Discrete Fourier Transform computation?

f) Determine the discrete-time frequency response $H_d(e^{j\omega})$ from the given specifications of continuous-time Linear Time Invariant (LTI) low pass filter H(jΩ), with sampling time T = 10^{-4} second.

$$0.99 \le |H(j\Omega)| \le 1.01, \quad \text{for} \quad |\Omega| \le 2\pi(1000)$$

$$|H(j\Omega)| \le 0.01, \quad \text{for} \quad |\Omega| \ge 2\pi(1100)$$

g) Prove following properties of the z-transform:

i) Linearity

ii) $x^*[n] \xleftrightarrow{z} X^*(z^*)$

(7x4)

2.

a) Explain two methods to design discrete time infinite impulse response from continuous time filter.

b) Design direct form-I and direct form- II realization of the given system function:

$$H(z)=\frac{1+\frac{1}{5}z^{-1}}{\left(1-\frac{1}{2}z^{-1}+\frac{1}{3}z^{-2}\right)\left(1+\frac{1}{4}z^{-1}\right)}$$

c) A discrete-time causal LTI system has the system function:

$$H(z) = \frac{(1+0.2z^{-1})(1-9z^{-2})}{1+0.81z^{-2}}$$

i) Is this system stable?

ii) Find expression for a minimum phase system $H_1(z)$ and an all pass system $H_{ap}(z)$ such that $H(z) = H_1(z)\, H_{ap}(z)$.

(6+6+6)

3.

a) Determine the z-transform and its Region Of Convergence (ROC) of the following sequence:

i) $x[n] = n\,a^n u[n]$ (using differentiation in z domain property)

ii) $\left(\frac{1}{2}\right)^n \left(u[n] - u[n-10]\right)$ (using time shifting property)

b) Determine the Fourier transform *X(w)* of the signal $x[n] = \{1, 2, \underset{\uparrow}{3}, 2, 1, 0\}$ and compute the 6-point Discrete Fourier Transform (DFT) *V(k)* of the signal $v[n] = \{\underset{\uparrow}{3}, 2, 1, 0, 1, 2\}$. Is there any relation between *X(w)* and *V(k)*? Explain.

c) A continuous-time low pass filter with frequency response $H_c(j\omega)$ is given. Detail specifications are:

$$1-\delta_1 \le |H_c(j\Omega)| \le 1+\delta_1, \qquad |\Omega| \le \Omega_p,$$

$$|H_c(j\Omega)| \le \delta_2, \qquad |\Omega| \ge \Omega_s.$$

and a set of discrete-time low pass filters can be obtained from $H_c(s)$ by using the bilinear transformation:

$$H(z) = H_c(s)\Big|_{s=(2/T_d)[(1-z^{-1})/(1+z^{-1})]}$$, with T_d as variable.

i) Assuming that Ω_p is fixed, find the value of T_d such that the corresponding pass band cutoff frequency for the discrete-time system is $\omega_p = \pi/2$.

ii) With Ω_p fixed, sketch ω_p as a function of $0 < T_d < \infty$.

iii) With both Ω_p and Ω_s fixed, sketch the transition region $\Delta\omega = (\omega_s - \omega_p)$ as a function of $0 < T_d < \infty$.

(6+6+6)

4.

a) Derive and draw 2nd order structure of the lattice filter and comment on significance of lattice filter in Speech Processing.

b) Determine the lattice coefficients corresponding to the Finite Impulse Response (FIR) filter with system function:

$$H(z) = A_3(z) = 1 + \frac{13}{24}z^{-1} + \frac{5}{8}z^{-2} + \frac{1}{3}z^{-3}$$

c) Write short note on "Time dependent Fourier analysis of Radar signals".

(7+7+4)

5.

a) Determine the impulse response and the step response of the causal system, y[n]. Plot the pole-zero patterns and determine stability of the same.

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n]$$

b) i) Construct a flow graph *or* butterfly diagram for a 16-point radix-2 decimation-in-time Fast Fourier Transform (FFT) algorithm.

Label: (1) All multipliers in terms of powers of W_{16}.

(2) Any branch transmittances those are equal to –1.

(3) The input and output nodes with appropriate values of the input and DFT sequences, respectively.

ii) Determine the number of real multiplications and number of real additions to implement the flow graph.

(9+9)

6.

a) Draw a parallel and a cascade realization of the system:

$$H(z) = \frac{10(1-\frac{1}{2}z^{-1})(1-\frac{2}{3}z^{-1})(1+2z^{-1})}{(1-\frac{3}{4}z^{-1})(1-\frac{1}{8}z^{-1})[(1-(\frac{1}{2}+j\frac{1}{2})z^{-1})][(1-(\frac{1}{2}-j\frac{1}{2})z^{-1})]}$$

b) Linear prediction is an important concept in digital signal processing in practical applications. Realize the lattice filter structure with the problem of linearly predicting the value of a stationary random process either forward in time or backward in time.

c) Consider the signal $x[n] = \{-1, 2, -\underset{\uparrow}{3}, 2, -1\}$ with Fourier transform *X(ω)*. Compute the following quantities, without explicitly computing *X(ω)*:

i) $X(0)$

ii) $\angle X(w)$

iii) $\int_{-\pi}^{\pi} X(w)dw$

iv) $X(\pi)$

(7+7+4)

7.

a) Explain how does 2-Dimension signal processing helpful in processing images.

b) Determine the sequence $x_3(n)$ corresponding to the circular convolution of the sequences $x_1[n]$ and $x_2[n]$. Verify this property of DFT using mathematical definitions of the DFT and IDFT. For given $x_1[n] = \{\underset{\uparrow}{2}, 1, 2, 1\}$ and $x_2[n] = \{\underset{\uparrow}{1}, 2, 3, 4\}$.

c) Explain the data compression strategies in terms of lossless and lossy algorithms.

(7+7+4)