C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) In a certain recruitment test there are multiple-choice questions. There are 4 possible answers to each question, of which one is correct. The possibility that a student knows the answer to a question is 90%. If the student answer the question and he get it correct then, what is the probability the he had guessed it but actually do not know the answer.

b) Find the entropy of the following distribution

events	x_1	x_2	x_3	x_4
probability	1/2	1/4	1/8	1/8

c) Find the conditional probability distribution of X when Y=2 for the following bivariate distribution

Y \ X	0	1
1	4/15	7/15
2	1/15	3/15

d) The mean arrival rate at a milk booth is one customer every 4 minutes and the mean service time is 2½ minutes. If the waiting cost is Rs.5 per unit per minute and the cost of servicing one unit is Rs. 4, then find the minimum cost service rate.

e) Write the dual of the following linear programming problem:

Min $z = x_1 - 3x_2$

$3x_1 - x_2 \leq 7$

$2x_1 - 4x_2 \geq 12$

$x_1 \geq 0$, x_2 – unrestricted in sign.

f) Given that $\mathcal{L}(\sin 2t) = 2/(s^2+4)$, find the Laplace transform of $\sin^2 t$.

g) A book of 525 pages contains 42 typographical errors. If these errors are Poisson distributed throughout the book, what is the probability that 10 pages, selected at random will have at the most 2 errors?

(7x4)

2.

a) Problems arrive at a computing centre in Poisson fashion at an average rate of five per day. The rules of the computing centre are that any man waiting to get his problem solved must aid the man whose problem is being solved. If the time to solve a problem with one man has an exponential distribution with mean time 1/3 day, and if the average solving time is inversely proportional to the number of people working on the problem, find the expected time for a person to get his problem solved by entering the line.

b) If the results of an investigation by an expert on a fire accident are summarized in the following:

i) Probability (there could have been a short circuit) = 0.8

ii) Probability (LPG explosion cylinder) = 0.2

iii) Chance of fire accidents is 30% given a short circuit and 95% given on LPG explosion.

Using Bayes' theorem, find the most probable cause of fire?

(9+9)

3.

a) The occurrence of rain in a city on a day depends upon whether or not it rained on the previous day. If it rained on the previous day, the rain distribution is

Event :	No rain	1 cm rain	2 cm rain	3 cm rain
Probability:	0.50	0.25	0.15	0.10

If it did not rain on the previous day, the rain distribution is

Event:	No rain	1 cm rain	2 cm rain	3 cm rain
Probability:	0.75	0.15	0.06	0.04

Assume that for the first day of the simulation it had not rained the day before. Stimulate the city's weather for 5 days and determine total days without rain as well as the total rainfall during the period. Use the following random numbers 67, 63, 39, 55, 29 for simulation.

b) A doctor recommends a patient to go on a particular diet for two weeks and there is equally likelihood for the patient to lose his weight between 2kg and 4kg. What is the average amount the patient is expected to lose on this diet?

(10+8)

4.

a) There are two market products of brands A and B respectively. Let each of these brands have exactly 50% of the total market in same period and let the market be of a fixed size. The transition matrix is given below:

$$\begin{array}{cc} & \textit{To} \\ & \begin{array}{cc} A & B \end{array} \\ \textit{From} \begin{array}{c} A \\ B \end{array} & \begin{bmatrix} 0.9 & 0.1 \\ 0.5 & 0.5 \end{bmatrix} \end{array}$$

If the initial market share break down is 50% for each brand, then determine their market shares in the steady state at the beginning of second period.

b) A transmitter has an alphabet consisting of five letters [x_1, x_2, x_3, x_4, x_5] and the receiver has an alphabet of four letters [y_1, y_2, y_3, y_4]. The joint probabilities for the communication are given as follows:

	y_1	y_2	y_3	y_4
x_1	0.25	0.00	0.00	0.00
x_2	0.10	0.30	0.00	0.00
x_3	0.00	0.05	0.10	0.00
x_4	0.00	0.00	0.05	0.10
x_5	0.00	0.00	0.05	0.00

Determine the marginal entropies for this channel.

(9+9)

5.

a) Use the method of Laplace transform solve the initial value problem.

$$y'' + 3y' + 2y = e^{-t}$$

with initial values $y(0) = -1, y'(0) = 1$.

b) Using branch and bound method solve the following mixed integer linear programming problem:

$$\text{Max } Z = 7x_1 + 9x_2$$
$$-x_1 + 3x_2 \le 6$$
$$7x_1 + x_2 \le 35$$
$$x_2 \le 7$$
$x_1, x_2 \ge 0$, only x_1 is an integer.

(9+9)

6.

a) Solve the following linear programming problem using the simplex method:

$$\text{Max } Z = 4x_1 + 10x_2$$
$$2x_1 + x_2 \le 10$$
$$2x_1 + 5x_2 \le 20$$
$$2x_1 + 3x_2 \le 10$$
$$x_1, x_2 \ge 0$$

b) Find the Fourier series of the function:

$f(x) = x + \pi, \; -\pi < x < \pi$

$f(x + 2\pi) = f(x)$

(9+9)

7.

a) A manufacturing concern produces a product consisting of two raw materials A_1 and A_2. The production function is estimated as

$$Z = f(x_1, x_2) = 3.6x_1 - 0.4x^2_1 + 1.6x_2 - 0.2x^2_2$$

where Z represents the quantity (in tons) of the product produced and x_1 and x_2 designated the input amounts of raw materials A_1 and A_2. The company has a constraint $2x_1 + x_2 = 10$. Determine how much input amounts x_1 and x_2 be decided so as to maximize the production output.

b) A random variable X assumes any positive integral value n with a probability proportional to $\frac{1}{3^n}$. Find the expectation of X.

(10+8)