NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours   **Total Marks: 100**

1.

a) Obtain the matrix AB and BA for the matrices A=$\begin{bmatrix} -4 & 3 \\ 7 & -2 \\ 9 & 0 \end{bmatrix}$ and B=$\begin{bmatrix} -2 & 5 & 3 \\ 1 & 6 & -1 \end{bmatrix}$.

b) Evaluate $\displaystyle \lim_{x \to \frac{\pi}{2}} (\sec x - \tan x)$.

c) Find the equation of the circle whose centre is $(2,3)$ and which passes through the intersection of the straight lines $3x - 2y - 1 = 0$ and $4x + y - 27 = 0$.

d) Test the convergence of the infinite series $\displaystyle \sum_{n=1}^{\infty} \frac{2n+1}{(n+1)^2}$.

e) Find the value of b for which the roots of the quadratic equation $4x^2 + bx + 9 = 0$ are double the roots of the quadratic equation $2x^2 - 5x - 3 = 0$.

f) Solve the following system of linear equations by the Cramer's rule

$$x + 2y = 9$$
$$2x - 3y = 4,$$

showing the detailed working of the rule.

g) The mean and the standard deviation of binomial distribution are 6 and $\sqrt{2}$ respectively. Find n, p, q and describe the distribution function. Here, all notations are taken in usual sense of binomial distribution.

(7x4)

2.

a) Evaluate $\displaystyle \int_{-4}^{0} \sqrt{16 - x^2} \, dx$.

b) Find the critical points of the function $f(x) = x^3 - 12x - 5$ and also identify the intervals in which $f(x)$ is increasing or decreasing.

c) Find the equation of an ellipse centered at origin and having axes as co-ordinate axes with latus rectum has length 4 and the distance between its two foci is $4\sqrt{2}$.

(6+6+6)

3.

a) Find the rank of the matrix $A = \begin{bmatrix} 1 & 5 & 4 \\ 0 & 3 & 2 \\ 2 & 13 & 10 \end{bmatrix}$.

b) Find the regression line of y on x for the following data:

x	1	4	2	3	5
y	3	1	2	5	4

(8+10)

4.

a) Find the sum of the series $\displaystyle\sum_{n=1}^{\infty} \frac{3^{n-1}-1}{6^{n-1}}$.

b) Find the point c in the Lagrange Mean Value Theorem for the function $f(x) = 1 - x^2$ in the interval $[0,2]$.

c) Evaluate the following integrals:

i) $\displaystyle\int \frac{x^5 - 1}{x - 1} dx$

ii) $\displaystyle\int \frac{x+3}{x^2 - 3x + 2} dx$

(5+5+8)

5.

a) Find the Taylor series expansion of $f(x) = \dfrac{1}{x}$ at the point 2.

b) Test the convergence the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{8 \tan^{-1} n}{n^2 + 1}$.

c) Find the value of λ for which the three vectors $4\hat{i} - \hat{j} + 3\hat{k}, \ -\hat{i} + \lambda\hat{j} + 2\hat{k}, \ -3\hat{i} + 2\hat{j} - \hat{k}$ are coplanar.

(8+6+4)

6.

a) From the following data calculate the missing frequency when mean is 34

Marks	0-10	10-20	20-30	30-40	40-50	50-60
No. of Students	5	15	20	?	20	10

b) If arithmetic mean and geometric mean of two positive numbers are 10 and 8 respectively, then find those two numbers.

c) Suppose that a book of 600 pages contains 40 printing errors, which are randomly distributed throughout the book; the number of errors per page has a Poisson distribution. What is the probability that 10 pages selected at random will be free of errors?

(6+6+6)

7.

a) With usual notations, find p of binomial distribution, if $n = 6$ and $9\,P(X = 4) = P(X = 2)$.

b) Find the eigen value and eigen vectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

(8+10)