BE10-R4: APPLIED OPERATIONS RESEARCH

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) A company has three operational departments (weaving, processing, packing) with capacity to produce three different types of clothes, namely, suiting, shirting and woollens. The profits are Rs. 20, Rs. 40, Rs. 30, per meter respectively. One meter of suiting requires 3 minutes of weaving, 2 minutes of processing and 1 minute of packing. One meter of shirting requires 4 minutes of weaving, 1 minute of processing and 3 minutes of packing, while one meter of woollen requires 3 minutes in each department. Total run time of each department in a day is 8 hrs., 10 hrs., 7 hrs., respectively. Formulate the problem as a linear program to maximize the profit.

b) In a factory there are five jobs to perform, each of which should go through two machines A and B in the order A and B, respectively. The processing time (in hours) for the jobs is given below:

	J_1	J_2	J_3	J_4	J_5
Machine A	7	2	10	4	11
Machine B	3	7	8	9	5

Determine the sequence for performing the jobs that could minimize the total elapsed time. Also find the minimum value of the total elapsed time.

c) Determine the optimal strategies for player I and player II, and the optimal value of the game whose pay-off matrix is given by

$$\begin{array}{c} \\ Player\ I \end{array} \begin{array}{c} \begin{array}{cccc} Player\ II & & & \end{array} \\ \begin{pmatrix} 0 & 0 & -2 & 5 \\ 1 & 2 & 3 & 1 \\ -3 & -4 & 12 & 0 \\ 1 & 3 & -2 & 4 \end{pmatrix} \end{array}.$$

d) Draw a network diagram for the following activities from A to L:

i) A, B, C can start simultaneously
ii) A and B precede D
iii) B precedes E, F and H
iv) F and C precede G
v) E and H precede I and J
vi) C, D, F and J precede K
vii) K precedes L
viii) I, G and L are terminal activities

e) Write the dual of the following LPP:

Max Z = $x - 5y + 9w$

Subject to $5x - 2y + 3w \geq 7$

$7x + 3y + w = 3$

$x + y + w \leq 4$

$x, y \geq 0$, w is unrestricted in sign.

f) The past records of a company indicate that each of the 5 high ended printers need repair after about 200 hours of use. The breakdowns are Poisson distributed. The one technician on duty can service a printer in an average 2 hours following exponential distribution. What is the average length of the queue?

g) A refrigerator manufacturer estimated a demand of 10000 units for the year. It costs about Rs. 1000 to set up the manufacturing process, and the carrying cost is about Rs. 50 per unit per year. When the production process has been set up, 80 refrigerators units can be manufactured daily. The demand during the production period is usually 60 units per day. How many refrigeration units should be manufactured in each batch?

(7x4)

2.

a) Use the Big-M method to solve the linear programming problem

$$\text{Max } Z = -x_1 + 3x_2$$
$$\text{Subject to } x_1 + 2x_2 \geq 2$$
$$3x_1 + x_2 \leq 3$$
$$x_1 \leq 4$$
$$x_1, x_2 \geq 0.$$

b) Find the job sequence that minimizes the total elapsed time required to complete five jobs on three machines M1, M2, M3.

Jobs	A	B	C	D	E
Time on M1	7	9	5	6	10
Time on M2	4	5	1	2	3
Time on M3	3	8	7	5	4

Also determine the total elapsed time and idle time of the three machines.

(9+9)

3.

a) A project has the following time schedule:

Activity	1-2	1-3	2-3	3-4	3-5	2-4	2-6	4-5	4-6	5-6
T_0	2	1	0	3	3	2	5	4	2	2
T_p	8	7	0	10	9	6	12	10	8	6
T_m	5	4	0	5	6	4	7	6	5	4

Construct the network. Find the critical path and expected time duration and also variance of the time duration.

b) Solve the following game by graphical method.

$$\begin{matrix} & \text{Player II} \\ \text{Player I} & \end{matrix} \begin{pmatrix} 4 & -1 & 0 \\ -1 & 4 & 2 \end{pmatrix}.$$

(9+9)

4.

a) Find the optimal transportation schedule and an optimal transportation cost of the following minimizing transportation problem:

	D_1	D_2	D_3	D_4	Availability
O_1	9	16	15	9	15
O_2	2	1	3	5	25
O_3	6	4	7	3	20
Requirement	10	15	25	10	

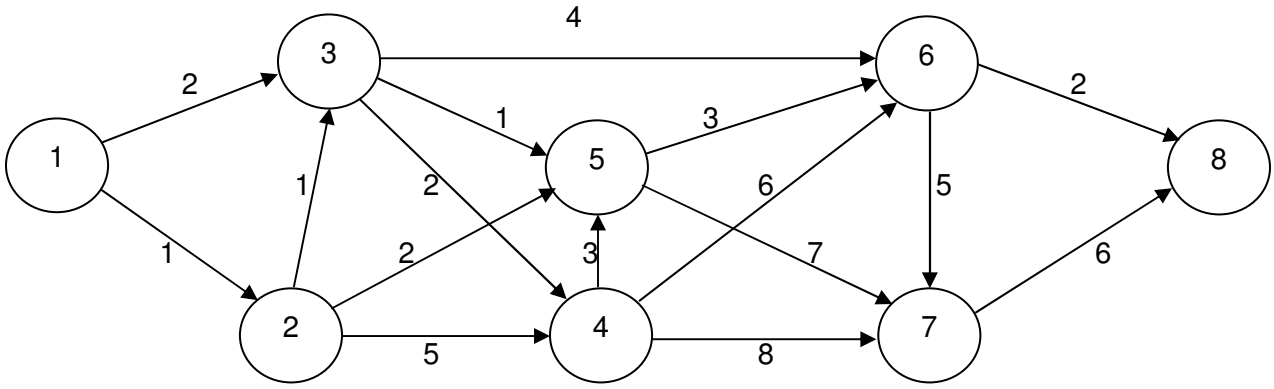
b) The network in the following figure represents the distances in Km. between various cities *i, i=1, 2, …, 8*. Find the shortest routes between the following pairs of cities:

i) City 1 and city 8.

ii) City 1 and city 6.

iii) City 4 and city 8.

iv) City 2 and city 6.



(9+9)

5.

a) Solve the following integer programming problem using the branch and bound method.

Min Z = $3x_1 + 8x_2$

Subject to $3x_1 - x_2 \leq 12$

$3x_1 + 11x_2 \leq 66$

$x_1\ x_2 \geq 0$

x_1, x_2 are integers.

b) A company has a chain of hotels operating in a city. The company uses a toll free telephone number to take reservation for any of these hotels. The average time to handle the call is 3 minutes and on an average 12 calls are received per hour. The probability distribution of arrival is not known. Over a period of time it is determined that an average caller spends 6 minutes on receiving a call. Find the (i) average time in the queue, (ii) an average time in the system, (iii) an average number in the system.

(10+8)

6.

a) Heads of four departments are capable of performing any of the four administrative duties assigned to them by the Dean. The estimated time (in hour) taken by the heads to perform various tasks is given in the following table. How should the task be allocated, one to each head, so as to minimize the total time for completing all tasks?

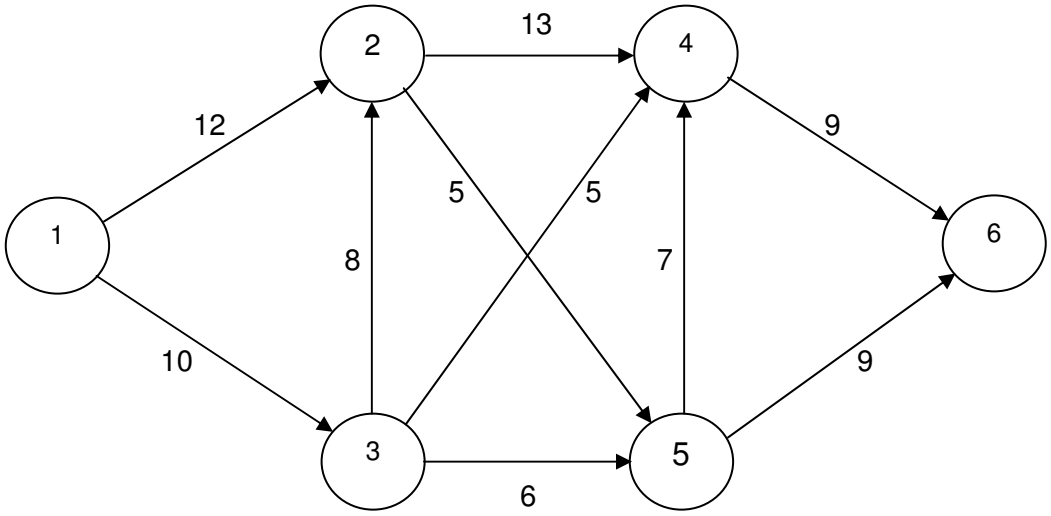
Time (in hrs.)	Task 1	Task 2	Task 3	Task 4
Prof 1	20	28	19	13
Prof 2	15	30	16	28
Prof 3	40	21	20	17
Prof 4	21	28	26	12

b) A stationary manufacturer has to supply 500 pens per day. He finds that when he starts a production run, he can produce 400 pens per day. The cost of holding a pen in a stock for one year is Rs. 5 and the set up cost of a production run is Rs. 180. Find, how frequently the production run should be made?

(10+8)

7.

a) Find the shortest path from node 1 to node 6 in the following network using Dijkstra's algorithm. The entries along the arcs represent the distances.



b) Consider the function

$$f(x_1, x_2, x_3) = x_1 + 2x_3 + x_2 x_3 - x_1^2 - x_2^2 - x_3^2$$

Compute the stationary point defined by vector $x_0=(x_{01}, x_{02}, x_{03})$, and show that the Hessian matrix is negative definite at x_0.

(9+9)