No. of Printed Pages : **3**

Sl. No.

C3-R4 : MATHEMATICAL METHODS FOR COMPUTING

NOTE :
1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours **Total Marks : 100**

1. (a) The amount of time in hours that a computer functions before breaking down is a continuous random variable with probability density function given by

$$f(x) = \begin{cases} \lambda e^{-\frac{x}{100}} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

What is the probability that

(i) a computer will function between 50 and 150 hours before breaking down ?

(ii) it will function for fewer than 100 hours ?

(b) A markov chain X_1, X_2, ... on states 0, 1, 2 has the transition probability matrix

$$P = \begin{array}{c c} & \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \end{array} & \begin{bmatrix} 0.1 & 0.2 & 0.7 \\ 0.2 & 0.2 & 0.6 \\ 0.6 & 0.1 & 0.3 \end{bmatrix} \end{array}$$

Compute the two steps transition probability matrix P^2. What is $\Pr\{X_3 = 1 | X_1 = 0\}$?

(c) Consider a cable modem which is used to transmit 8-bit characters and which has a capacity of 4 megabits per second (Mbps). Given that traffic arrives according to Poisson Distribution at a rate of 450,000 cps, compute the mean number of characters in the system, and the mean number of characters waiting to be transmitted.

(d) Find the Fourier transform of

$$f(x) = \begin{cases} 1 - x^2, & |x| \leq 1 \\ 0, & |x| > 1 \end{cases}$$

Hence evaluate $\int_0^\infty \frac{x\cos x - \sin x}{x^3} \cos\left(\frac{x}{2}\right) dx$.

(e) Find the dual of the following linear programming problem

Minimize $Z = 4x_1 + 3x_2$

Subject to $5x_1 + x_2 \geq 11$

$2x_1 + x_2 \geq 8$

$x_1 + 2x_2 \geq 7$

$x_1 \geq 0, x_2 \geq 0$

(f) Determine $L^{-1}\left\{\dfrac{4s^2 - 5s + 6}{(s+1)(s^2+4)}\right\}$

(g) Write the algorithm to simulate random variable X such that $p_1 = 0.20$, $p_2 = 0.15$, $p_3 = 0.25$, $p_4 = 0.40$ where $p_j = P\{X = j\}$. **(7x4)**

2. (a) A pair of fair dice is rolled. Let $X = \begin{cases} 1, \text{ if the sum of the dice is 6} \\ 0, \text{ otherwise} \end{cases}$

and let Y equal the values of the first die. Compute (i) $H(Y)$ (ii) $H(X,Y)$

(b) Consider a discrete memoryless system has four symbols x_1, x_2, x_3, x_4 with $p(x_1) = \frac{1}{2}$, $p(x_2) = \frac{1}{4}$ and $p(x_3) = p(x_4) = \frac{1}{8}$. Construct a Shannon-Fano code for X; show that this code has the optimum property that $n_i = I(x_i)$ and that the code efficiency is 100 percent. **(9+9)**

3. (a) Find the Laplace transform of (i) t cos at (ii) t^2 sin at

(b) If X and Y are independent binomial random variable with identical parameters n And p, calculate the conditional expected value $E(X|X+Y=m)$. **(9+9)**

4. (a) Find the Laplace transform of the function $f(t)$ defined by

$$f(t) = 3 \quad 0 < t < 2$$
$$= 0 \quad 2 < t < 4$$
$$f(t+4) = f(t)$$

(b) Find the Fourier coefficients corresponding to the function $F(x) = \begin{cases} 0 & -5 < x < 0 \\ 3 & 0 < x < 5 \end{cases}$

Period = 10. Write the corresponding Fourier series. **(9+9)**

5. (a) Let Z_1, Z_2, ... be independent and identically distributed random variables with $P(Z_n = 1) = p$ and $P(Z_n = -1) = q = 1 - p$ for all n and $X_0 = 0$. Find the mean and variance of $\{X_n, n = 1, 2, \ldots\}$

(b) Consider a two-state Markov chain with the transition probability matrix

$$P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}, 0 < a < 1, 0 < b < 1$$

Show that the n-step transition probability matrix P^n is given by

$$P^n = \frac{1}{a+b}\left\{\begin{bmatrix} b & a \\ b & a \end{bmatrix} + (1-a-b)^n \begin{bmatrix} a & -a \\ -b & b \end{bmatrix}\right\}$$ **(9+9)**

6. (a) Consider an $M/M/1/\infty$ queuing system with an arrival rate of 3 per second and a service rate of 5 per second, operating under equilibrium.

(i) A customer A enters the system when the server is free. What is the probability that another customer enters before customer A leaves the system ?

(ii) A customer A enters the system when the number of customers in the system is 3. The exact service time requirement of customer A is known to be 0.4 second. What is the expected response time of customer A ?

(iii) Determine the probability that the number of customers in the system does not change during a time interval of 0.1 second.

(b) Solve the following linear programming problem using simplex method.

Maximize $Z = -8x_1 + 3x_2 - 6x_3$

Subject to $x_1 - 3x_2 + 5x_3 = 4$

$5x_1 + 3x_2 - 4x_3 \geq 6$

$x_1, x_2, x_3 \geq 0$ **(9+9)**

7. (a) Solve the following linear programming using branch and bound method

Maximize $Z = 10x_1 + x_2$

Subject to $2x_1 + 5x_2 \leq 11$

$x_1, x_2 \geq 0$, x_1 and x_2 are integral

(b) Solve using Kuhn-Tuker conditions

Maximize $Z = 3x_1 + x_2$

Subject to $x_1^2 + x_2^2 \leq 5$

$x_1 - x_2 \leq 1$

$x_1 \geq 0, x_2 \geq 0$ **(9+9)**

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