Sl. No.

C0-R4.B1 : ELEMENTS OF MATHEMATICAL SCIENCES

NOTE :
1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours   **Total Marks : 100**

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1. (a) The chance that Doctor P will diagnose disease Q correctly is 60%. The chance that a patient will die by his treatment after the correct diagnosis is 40% and the chance of death after the wrong diagnosis is 70%. A patient of Doctor P who had disease Q died. The probability that his disease was diagnosed ?

(b) Using determinants, find the area of the triangle with vertices $(-2, -3)$, $(3, 2)$ and $(-1, -8)$.

(c) Find $\dfrac{dy}{dx}$, if $y = \log\left[a^{2x}\left(\dfrac{x+3}{2x-1}\right)^{\frac{3}{2}}\right]$.

(d) Test the convergence of the infinite series $\displaystyle\sum_{n=1}^{\infty}\left(\dfrac{1}{n} - \log\left(\dfrac{n+1}{n}\right)\right)$.

(e) A company is planning to improve plant safety. For this, the accident data for the last 50 weeks was compiled. These data are grouped into the frequency distribution as shown below. Calculate the arithmetic mean of the number of accidents per week.

Number of accidents (x)	0 - 4	5 - 9	10 - 14	15 - 19	20 - 24
Number of weeks (f)	5	22	13	8	2

(f) Show that $u(x) = (\sin t)i + (\cos t)j + \sqrt{3}k$ has constant length and is orthogonal to its derivative.

(g) Evaluate $\displaystyle\lim_{x\to\infty}\left(\dfrac{ax+1}{ax-1}\right)^{x}$.

[7x4]

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2. (a) Find the eigen values and eigen vectors for the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.

(b) Evaluate $\displaystyle\int_0^{\frac{\pi}{2}} \frac{\cos x}{(2 + \sin x)(1 + \sin x)}\, dx$.

(c) In a factory which manufacture bolts, machines A, B and C manufacture respectively 25%, 35% and 40% of the bolts. Of their outputs, 6, 4 and 2 percent are respectively defective bolts. A bolts is drawn at a random from the product and is found to be defective. What is the probability that it is manufactured by machine B ?

[6+6+6]

3. (a) Two hundred digits were chosen at random from a set of tables. The frequencies of the digits are as below :

Digit	0	1	2	3	4	5	6	7	8	9
Frequency	18	19	23	21	16	25	21	20	22	15

Use chi-square test to assess the correctness of the hypothesis that the digits were distributed in an equal manner in the tables from which they are chosen.

(b) The life time of a certain kind of batteries has a mean life of 400 hours and standard deviation as 45 hours. Assuming the distribution of life to be normal, find :

(i) the percentage of batteries with a life time of at least 470 hours

(ii) the proportion of batteries with a lifetime between 385 and 415 hours

(iii) the minimum life of the best 5% of batteries

[8+10]

4. (a) The following is the record goals scored by Team A in a football season :

No. of goals scored by Team A in match	0	1	2	3	4
No. of matches played in the month	1	9	7	5	3

For Team B, the average number of goals scored per match was 2.5 with standard deviation of 1.25 goals. Find which team may be considered more consistent.

(b) Find a Cartesian equation for the hyperbola centered at the origin that has the focus at (3, 0) and the line $x = 1$ as the corresponding directrix.

(c) Test the convergence of the infinite series $\displaystyle\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{\log(n+1)}$.

[6+6+6]

5. (a) If $0 < a < b$, prove that $\left(1 - \dfrac{a}{b}\right) < \log \dfrac{b}{a} < \left(\dfrac{b}{a} - 1\right)$. Hence prove that $\dfrac{1}{6} < \log (1.2) < \dfrac{1}{5}$

and $\dfrac{1}{2} < \log 2 < 1$.

(b) Solve the following system for x, y and z.

$$\dfrac{-1}{x} + \dfrac{3}{y} + \dfrac{4}{z} = 30$$

$$\dfrac{3}{x} + \dfrac{2}{y} - \dfrac{1}{z} = 9$$

$$\dfrac{2}{x} - \dfrac{1}{y} + \dfrac{2}{z} = 10$$

[9+9]

6. (a) The following data regarding the heights(y) and weights (x) of 100 college students are given :

$$\sum x = 15000, \sum x^2 = 2272500, \sum y = 6800, \sum y^2 = 463025, \sum xy = 1022250$$

Find the coefficient of correlation between height and weight and also the equation of regression of height and weight.

(b) Show that $\left[\text{diag}\,(\alpha,\, \beta,\, \gamma)\right]^{-1} = \text{diag}\left(\dfrac{1}{\alpha},\, \dfrac{1}{\beta},\, \dfrac{1}{\gamma}\right)$, if $\alpha\beta\gamma \neq 0$.

[10+8]

7. (a) If two cards are drawn from a pack of 52 cards which are diamonds, using Poisson distribution, find the probability of getting two diamonds at least 3 times in 51 consecutive trials of two cards drawing each time.

(b) In a partially destroyed laboratory record of an analysis of correlation data, the following results are legible :

Variance of $x = 9$

Regression equations : $8x - 10y + 66 = 6$

$\qquad\qquad\qquad\qquad\qquad\quad 40x - 18y = 214$

What are :

(i) Means of x's and y's

(ii) the coefficient of correlation between x and y

(iii) the standard deviation of y ?

[9+9]

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