B3.2 - R4 : DISCRETE STRUCTURES

**NOTE :**
1. **Answer question 1 and any FOUR questions from 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time : 3 Hours   **Total Marks : 100**

1. (a)   Find the negation of statement "For all integers n, either n is even or n is odd."

(b)   In how many ways, can we select a team of 4 students from a given choice of 15 ?

(c)   State Lagrange's theorem.

(d)   What is the language accepted by the following finite automata ?
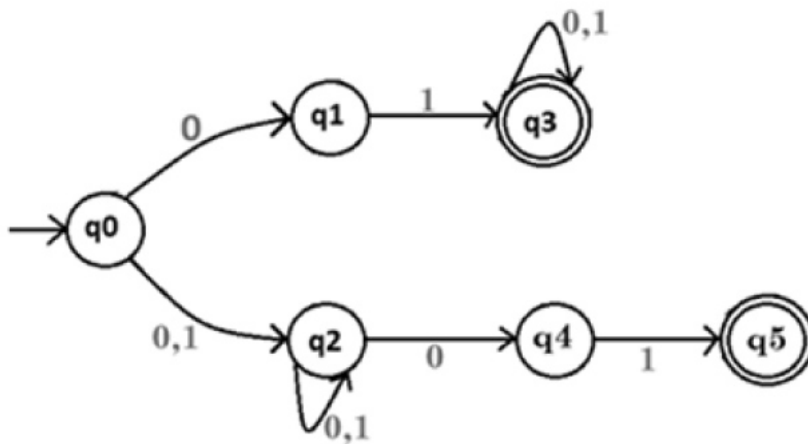
(e)   Let G is a simple graph with 40 edges and its complement G' has 38 edges. Find the number of vertices in the graph G.

(f)   Is it possible for a planar graph to have 6 vertices, 10 edges and 5 faces ? Explain.

(g)   What is recurrence for worst case of QuickSort and what is the time complexity in Worst case ?

                                                                                                                  (7x4)

2. (a)   Find all relations which exist between the following compound statements :

(i) $\sim a$

(ii) $a \wedge b$

(iii) $\sim a \vee b$

(iv) $\sim a \vee \sim b$

(v) $a \rightarrow b$

(b) Do the following statements have the same meaning as the statement, "If Jean is not in her room, then she is at the library."?

(i) If Jean is at the library, then she is not in her room.

(ii) If Jean is at her room, then she is not at the library.

(iii) If Jean is not at the library, then she is in her room.

(iv) Jean is in her room, or she is at the library. Are the statements above related to each other ? Explain.

(c) Verify that $(X + Y)(X + Z)(Y + Z) = (X + Y)(X + Z)$.

(6+6+6)

3. (a) Given $f(x, y, z) = yz(\bar{x} + y) + (\bar{z} + x)(y + z)$, put f into

(i) the disjunctive normal form (dnf)

(ii) the full disjunctive normal form

(iii) the sum-of-products form

(iv) the expanded sum-of-products

(b) Simplify the following Boolean expression using a Karnaugh map :

$E = x\bar{y}z + xz + x\bar{y}\,\bar{z} + (x+y)\bar{z}$

(c) Can a complete graph K5 be planar ? Justify your answer.

(6+6+6)

4. (a) Use the laws of Boolean algebra to simplify the following Boolean expression :

$[(A+B) + (A+C) + (A+D)] \bullet [A \bullet \sim B]$

Show the switching and gate representations of the resulting expression.

(b) A Sunday school class of 12 members is to be seated on seven chairs and a bench that accommodates five persons. In how many ways can the bench be occupied ?

(10+8)

5. **(a)** Find the minimum number of colors needed to paint the region of the following maps.
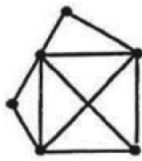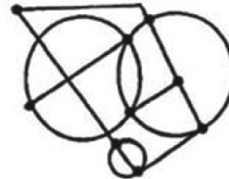

Fig. 1.   Fig. 2.   Fig. 3.

(b) Given different relations on a set, how do you find whether relations are reflexive, symmetric, anti-symmetric, or transitive ? Given set A = {1, 2, 3, 4, 5, 6, 7) and relations :

R1 = {(1, 1), (2, 2), (1, 2), (2, 3), (3, 2), (3, 3), (4, 4), (5, 5), (7, 6), (6, 7), (6, 6), (2, 1), (7, 7)}

R2 = {(1, 2), (2, 3), (1, 3), (1, 4), (2, 4)}

R3 = {(1, 4), (2, 5), (3, 7), (6, 6)}

R4 = {(2, 4), (4, 2), (4, 6), (2, 6), (6, 4), (6, 2), (2, 2), (4, 4), (6, 6)}

Determine which of the given relations from set A to it-self are :

(i)   Reflexive   (ii)   Symmetric

(iii) Transitive   (iv)   Anti-symmetric

**(c)** Demonstrate the insertion sort results for each insertion for the following initial array of elements.
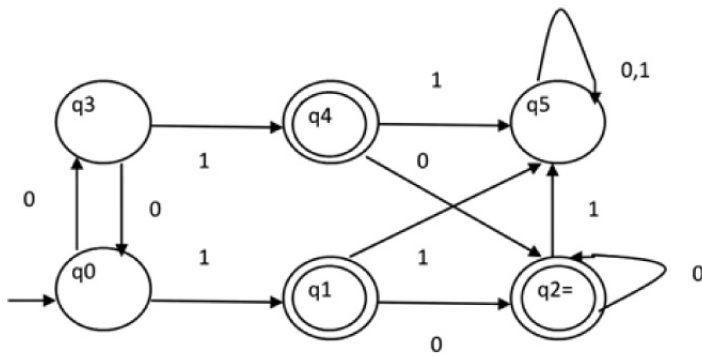
25, 6, 15, 12, 8, 34, 9, 18, 2   **(6+8+4)**

6. **(a)** Construct a DFA machine over input alphabet = {0, 1}, that accepts even number of 0's and even number of 1's.

(b) Differentiate a Mealy Machine from a Moore Machine.

(c) How many baseball teams of nine members can be chosen from among twelve boys, without regard to the position played by each member ?   **(8+6+4)**

7. (a) For the given graph use Prim's algorithm to find the Minimum Spanning tree.
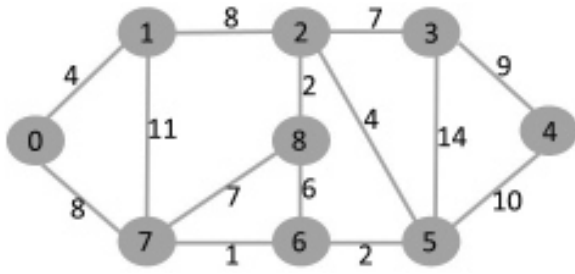
(b) Simplify the given Finite state Automata.

(9+9)

- o 0 o -