No. of Printed Pages : **2**

Sl. No.

B32 - R4 : DISCRETE STRUCTURES

NOTE :

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours | Total Marks: 100

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1. (a) For the function, $y=2x+1$, find the range when domain$=\{-3,-2,-1, 0, 1, 2, 3\}$.

(b) What probabilities should we assign to the outcomes H (heads) and T (tails) when a fair coin is flipped ? What probabilities should be assigned to these outcomes when the coin is biased so that heads comes up twice as often as tails ?

(c) Solve the recurrence relation, $a_n - 4a_{n-2} = 0$ for $n \geq 2$ with $a_0 = 1$ and $a_1 = 1$.

(d) Let $L = \{w \in \{a, b\}^* : w$ contains bba as a substring$\}$. Find a regular expression for $\{a, b\}^* - L$.

(e) Consider the relation, R on $A = \{1, 2, 3\}$ whose matrix $M_R = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{bmatrix}$ Compute the matrix M_{R^2}.

(f) A family of 4 brothers and 3 sisters is to be arranged for a photograph in one row. In how many ways they can be seated if

(i) all the sisters sit together.

(ii) no two sisters sit together.

(g) Find the greatest common divisor of 414 and 662 using the Euclidean algorithm.

(7x4)

2. (a) Let * be a binary operation defined on Q. Find which of the following binary operations are associative

(i) $a * b = a - b$ for $a, b \in Q$.

(ii) $a * b = \frac{ab}{4}$ for $a, b \in Q$.

(iii) $a * b = a - b + ab$ for $a, b \in Q$.

(iv) $a * b = ab^2$ for $a, b \in Q$

(b) Which of the following collections of subsets are partitions of the set of integers ?

(i) the set of even integers and the set of odd integers.

(ii) the set of positive integers and set of negative integers.

(iii) the set of integers divisible by 3, the set of integers leaving a remainder of 1 when divided by 3, and the set of integers leaving a remainder of 2 when divided by 3.

(iv) the set of integers less than -100, the set of integers with absolute value not exceeding 100, and the set of integers greater than 100.

(10+8)

3. (a) Show $\neg(p \rightarrow q)$ is equivalent to $p \wedge \neg q$.

(b) Use mathematical induction to prove that

$$1^2+2^2+3^2+....+n^2=\frac{1}{6}n(n+1)(2n+1)$$

(c) Use Karnaugh map to simplify the expression, $x'y'+yz+x'yz'$

(6+6+6)

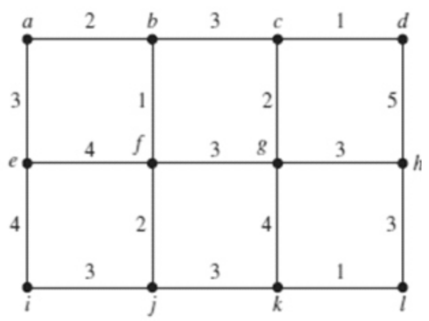
4. (a) Find the prime factorization of 7007.

(b) One card is drawn from a standard pack of 52 playing cards. Let A be the event that 'it is a red card' and B be the event that 'it is a court or face card.' What is the probability that the drawn card is red or face or both?

(c) Show that among any $n+1$ numbers, one can find 2 numbers so that their difference is divisible by n.

(6+6+6)

5. (a) Use Kruskal's algorithm to find a minimum spanning tree in the weighted graph



(b) Show that K_n has a Hamilton circuit whenever $n \geq 3$.

(9+9)

6. (a) Prove that running time $T(n) = n^3 + 20n$ is $\Omega(n^2)$.

(b) Prove that 'A simple graph is connected if and only if it has a spanning tree.'

(8+10)

7. (a) Find a Turing machine that recognizes the set $\{0^n1^n | n \geq 1\}$

(b) "Multiply" the sequence 1, 2, 3, 4, ...1, 2, 3, 4,... by the sequence 1, 2, 4, 8, 16,....

(9+9)

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