No. of Printed Pages : **3**

Sl. No.

B0-R4 : BASIC MATHEMATICS

NOTE :
1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours | Total Marks: 100

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1. (a) If $(x+yi)/i=(3+5i)$, where x and y are real, what is the value of $(x+yi)(x-yi)$?

(b) Evaluate the following definite integral $\int_4^9 \left(\sqrt{x} + \frac{1}{3\sqrt{x}}\right)dx$

(c) Write two different vectors having same magnitude.

(d) Without expansion, show that $\begin{vmatrix} 6 & 1 & 3 & 2 \\ -2 & 0 & 1 & 4 \\ 3 & 6 & 1 & 2 \\ -4 & 0 & 2 & 8 \end{vmatrix} = 0$.

(e) Find local maxima and minima of the function $f(x)=e^x+3e^{-x}$.

(f) Solve the differential equation $(x^2 y+2xy^2-y^3)dx-(2y^3-xy^2+x^3)dy=0$.

(g) If $\overrightarrow{PO} + \overrightarrow{OQ} = \overrightarrow{QO} + \overrightarrow{OR}$, then show that the points P, Q, R are collinear. **(7x4)**

2. (a) Show that $\begin{vmatrix} 1 & a & bc \\ 1 & b & ca \\ 1 & c & ab \end{vmatrix} = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$.

(b) Compute the cube roots of $z = -8$.

(c) Show that the function $y = (x+1) - \frac{1}{3}e^x$ is a solution to the first-order initial value problem $\frac{dy}{dx} = y - x,\ y(0) = \frac{2}{3}$.

(d) A rubber ball is thrown in the air. Its height at any instance of time is given by $h=3+14t-5t^2$ then what is the its maximum height ?

(e) Compute the limit $\lim_{x\to 0} \frac{\cos\left(x^4\right) - 1 + \frac{1}{2}x^8}{x^{16}}$ **(4+4+4+3+3)**

3. (a) Use Cramer's rule to solve the system

$-4x + 2y - 9z = 2$

$3x + 4y + z = 5$

$x - 3y + 2z = 8$

(b) If $\vec{a} = 2\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b} = -\hat{i}+2\hat{j} + \hat{k}$ and $\vec{c} = 3\hat{i}+\hat{j}$ are such that $\vec{a} + \lambda\vec{b}$ is perpendicular to $\vec{c}$, then find the value of λ.

(c) Find the first three terms in the Maclaurin series for $\sqrt{1-x+x^2}$.

(d) Determine all complex number z that satisfy the equation $z+3z' = 5-6i$ where z' is the complex conjugate of z.

(e) Find the derivative of the function $f(x) = \frac{x+1}{x-1}$ from first principle.

(4+4+4+2+4)

4. (a) Find the first four terms in the Taylor series for$(x-1)e^x$ near $x = 1$.

(b) Find the equation of the ellipse which passes through the point $(-3, 1)$ and has eccentricity $\sqrt{\frac{2}{5}}$ with x-axis as its major axis and centre at the origin.

(c) Find the value of k so that $y=e^{kx}$ is a solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 6y = 0$ and find the general solution.

(d) Find the equation of the circle which passes through the points (20, 3), (19, 8) and (2, −9). Also find its centre and radius.

(e) Solve the following differential equations $(x - 2y)\, dx + x\, dy = 0$

(4+4+4+4+2)

5. (a) The sum of the perimeter of a circle and square is k, where k is a constant. Prove that the sum of their area is minimum, when the side of square is double the radius of the circle.

(b) Find the eigen values and eigen vectors of matrix $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$

(c) If $f(x)$ is of the form $f(x) = a + bx + cx^2$, then show that

$$\int_0^1 f(x)dx = \frac{1}{6}\left\{f(0)+4f\left(\frac{1}{2}\right) + f(1)\right\}$$

(d) Find the differential equation of all the circles in the first quadrant which touch the coordinate axes.

(e) If D and E are the mid-points of sides AB and AC of a triangle ABC respectively, show that $\overrightarrow{BE}+\overrightarrow{DC} = \frac{3}{2}\overrightarrow{BC}$.

(4+4+4+4+2)

6. (a) Assume that the vectors $\vec{a}, \vec{b}$ and $\vec{c}$ given as $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$, $\vec{b} = b_1 \hat{i} + b_2 \hat{j} + b_3 \hat{k}$ and $\vec{c} = c_1 \hat{i} + c_2 \hat{j} + c_3 \hat{k}$, then show that $\vec{a} \times \left(\vec{b} + \vec{c} \right) = \vec{a} \times \vec{b} + \vec{a} \times \vec{c}$.

(b) Suppose a ball is thrown straight upward so that its height $f(x)$ (in feet) is given by the equation $f(x) = 96 + 64x - 16x^2$ where x is time (in seconds).

(i) Find the average velocity from $x = 1$ to $x = 1 + \text{h}$.

(ii) Find the instantaneous velocity at $x = 1$.

(c) If $|z_1 + z_2| = |z_1 - z_2|$, prove that the difference of amplitudes of z_1 and z_2 is $\pi/2$.

(d) Assume that a spherical drop evaporates at a rate proportional to its surface area. If its radius originally is 3 mm and 1 hour later has been reduced to 2 mm, find an expression for the radius of the rain drop at any time.

(e) Evaluate $\int_{\pi/4}^{\pi/2} \cos 2x \log \sin x \, dx$. **(4+4+3+4+3)**

7. (a) Check the convergence of the series : $\sum_{n=1}^{\infty} \left[\left(n^3 + 1 \right)^{1/3} - \text{n} \right]$

(b) Find all the asymptotes of the curve :

$$x^3 + 4x^2y + 5xy^2 + 2y^3 + 2x^2 + 4xy + 2y^2 - x - 9y + 1 = 0$$

(c) Solve the homogeneous linear differential equation : $3x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = x$ **(6+6+6)**

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