C4-R4: ADVANCED ALGORITHMS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) i) Prove that $A \in \Theta(B)$ A: $5n^2 + 100n$ B: $3n^2 + 2$, where A & B are the functions over n.

ii) $O(n^4)$, $O(1)$, $O(n)$, $O(n\log n)$, $O(n^2\log n)$, $\Omega(n^{0.5})$, $\Omega(n^2\log n)$, $O(\log_2 n)$.

Arrange the given terms in increasing order of their time complexity.

b) Solve the following recurrence using recurrence tree method $T(n) = 3T(n/4) + cn^2$.

c) State the Masters theorem for solving recurrence. Apply the Masters theorem to solve the following recurrence:

$T(n) = 9T(n/3) + n$

d) Define P, NP, NP-Complete and NP-Hard Problems.

e) Explain the elements of dynamic programming in brief.

f) What is Radix Sort? Sort the following elements in increasing order using radix sort technique.

459 254 472 534 649 239 432 654 477

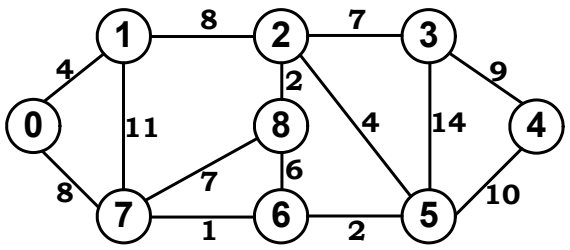
g) Show that the Travelling Salesman Problem is NP-Complete.

(7x4)

2.

a) Define and differentiate BFS and DFS. What are the advantages of one over other?

b) What is Minimum Spanning Tree? Mention few application of Minimum Spanning Tree. Apply Prim's Algorithm to find the Minimum Spanning Tree of following weighted graph:



c) Whether, Dynamic Programming is a top-down approach or it is a bottom up approach? Justify your answer with suitable example.

(8+5+5)

3.

a) How to find Longest Common Subsequence of two strings using Dynamic Programming Method? Find the Longest Common Subsequence of following two strings:

S1: president, S2: providence

b) Apply dynamic programming based approach to solve the O/I Knapsack problem for following:

Object Number	Weight	Profit
1	1	15
2	5	10
3	3	9
4	4	5

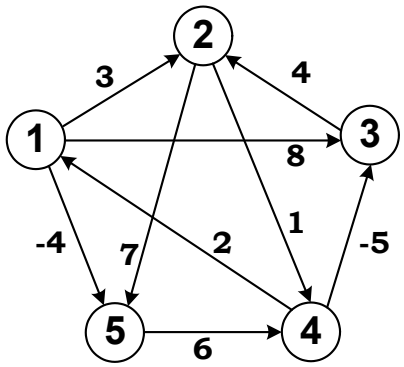
c) Sort the following list in increasing order using quick sort technique. Always consider first element as pivot element.

40, 20, 10, 80, 60, 50, 7, 30, 100

(8+6+4)

4.

a) Explain Floyd Warshall algorithm. Apply Floyd Warshall algorithm for computing the shortest path from all vertices to other vertices in following graph:



b) How many exact match and spurious hits does the Rabin-Karp matcher encounter in the text T = 31415926535 when looking for pattern P = 26? Take value of modulo q = 11.

c) What is Huffman Coding? Give advantages of Huffman coding. Create Huffman Tree from given input characters a, b, c, d, e and f whose frequencies are respectively 5, 9, 12, 13, 16 and 45.

(7+5+6)

5.

a) For the following six matrices find the order of parenthesization for optimal chain multiplication.

A1: 30 × 35, A2: 35 × 15, A3: 15 × 5, A4: 5 × 10, A5: 10 × 20, A6: 20 × 25.

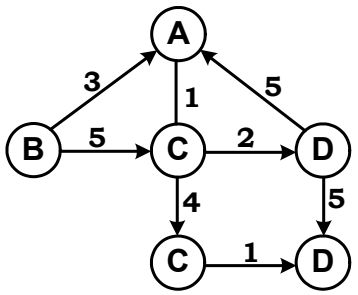
b) Define Binary Search? Write and analyze Binary Search Algorithm. Solve recurrence equation of binary search using Master theorem.

c) Write the 2-approximation algorithm for Vertex Cover Problem. Discuss, how it is 2-approximation?

(8+6+4)

6.

a) Explain Dijkstra's algorithm. Apply Dijkstra's algorithm to find shortest paths from source vertex B to all other vertices in following graph:



b) Write the algorithm to search a graph using Breadth First Search (BFS). Explain BFS using an example.

c) Is P = NP? Justify your answer.

(6+8+4)

7.

a) A sequence of n operations is performed on a data structure. The i^{th} operation costs i, if i is an exact power of 2, otherwise 1. Use aggregate analysis to determine the amortized cost per operation.

b) What is Starssen's algorithm? How Starssen's matrix multiplication can be done efficiently by using divide and conquer technique?

c) Show that for any real constants *a* and *b*, where $b > 0$, $(n+a)^b = \Theta(n^b)$

(5+8+5)