B0-R4: BASIC MATHEMATICS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If A = $\begin{bmatrix}1 & 9\\ 3 & 2\end{bmatrix}$ and B = $\begin{bmatrix}5 & 1\\ 7 & 9\end{bmatrix}$. Find matrix X such that 2A+3B+5X = 0 (Zero matrix).

b) Test the convergence of the series $1+\frac{2!}{2^2}+\frac{3!}{3^3}+\frac{4!}{4^4}+\ldots$

c) Evaluate the $\lim_{x \to 0}\left(\frac{e^x - e^{-x}}{x}\right)$.

d) Evaluate $\int_1^4 \frac{dx}{\sqrt{x^2+2x+5}}$.

e) Solve the differential equation: $\frac{dy}{dx} = y\cot 2x, y\left(\frac{\pi}{4}\right) = 2$.

f) Express $\frac{2-3\,i\sqrt{3}}{1+i}+\frac{2}{1-i}$ in the form of $a+ib$.

g) Find the workdone by the force $\overrightarrow{F} = 5i\text{-}3j\text{+}2k$ as its point of application moves from the point A(2, 1, 3) to the point B(4, -1, 5).

(7×4)

2.

a) Is matrix A=$\begin{bmatrix}-1 & 2 & 0\\ -1 & 1 & 1\\ 0 & 1 & 0\end{bmatrix}$ invertible? if yes, find A^{-1}. Show that A^{-1}=A^2.

b) Find the equation of the tangent and normal to the curve $y = \cos x$ at $x = \frac{\pi}{3}$.

(9+9)

3.

a) Find the Eigenvalues and Eigenvectors of the matrix

A=$\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$.

b) Evaluate the integral $\int \frac{dx}{2+\cos x-\sin x}$.

(10+8)

4.

a) Find the area of the region $\{(x,y) : y^2 \le 4x,\ 4x^2+4y^2 = 9\}$.

b) Solve the differential equation $y - x\frac{dy}{dx} = a\left(y^2+\frac{dy}{dx}\right)$.

(10+8)

5.

a) Find the points of local minima, if any, for the function
$f(x) = \sin 2x - x, \quad -\frac{\pi}{2} \le x \le \frac{\pi}{2}$.
Find also the local maximum and local minimum values.

b) Expand $log_e x$ in power of $(x-1)$ and hence evaluate $log_e 1.1$ correct to 4 decimal places.

(10+8)

6.

a) If $f(x) = \begin{cases} \frac{\sin}{x}, & x \neq 0 \\ 1 & x = 0 \end{cases}$,
find whether $f(x)$ is continuous at *x = 0*.

b) Find the equation of the plane through the points (1, 2, -3), (2, 3, -4) and perpendicular to the plane *x+y+z*+1=0**.**

(8+10)

7.

a) Find the equation of the ellipse whose focus is (0,1), directrix is *x+y=0* and eccentricity $e=\frac{1}{2}$.

b) Test the convergence of the series
$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \ldots + \frac{2^n-2}{2^n+1}x^{n-1} + \ldots . (x > 0).$

(9+9)