C3-R4: MATHEMATICAL METHODS FOR COMPUTING

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Events A and B are independent. Examine if the events $\bar{A}$ and B are independent?

b) A factory has four independent units A, B, C and D which produces 40%, 30%, 20% and 10% of identical items, respectively. The percentages of defective items produces by these units are 2%, 1%, 0.5% and 0.25% respectively. If an item is selected at random, find the probability that the item is defective.

c) If the probability density function of a continuous random variable is given by $f(x) = e^{-x}, 0 \le x < \infty$, find the mean and variance.

d) Convert the following L.P.P into standard form.

Maximize Z = $2x_1 + 3x_2 + 6x_3$

subject to $x_1 - 2x_2 \le 7$,
$3x_1 + 4x_2 - 6x_3 \ge 10$,
$4x_1 + 3x_3 \le 5$,
$x_1, x_2 \ge 0$.

e) Obtain the Laplace transform of the function

$$f(t) = \begin{cases} \sin t & \text{for } 0 < t < \pi \\ 0 & \text{for } t > \pi \end{cases}$$

f) Two players A and B play tennis games. Their chances of winning a game are in the ratio 3:2 respectively. Find A's chances of winning at least two chances out of four games played.

g) Find the Fourier series Expansion of the periodic function

$$f(x) = x, -\pi \le x \le \pi,$$
$$f(x + 2\pi) = f(x).$$

(7x4)

2.

a) Four boxes A, B, C, D contain fuses. The boxes contain 5000, 3000, 2000 and 1000 fuses respectively. The percentages of fuses in the boxes which are defective are 3%, 2%, 1% and 0.5% respectively. One fuse is selected at random arbitrarily from one of the boxes. It is found to be a defective fuse. Find the probability that it has come from the box D.

b) Show that the entropy of the following probability distribution is $2 - \frac{1}{2^{n-2}}$.

Events:	x_1	x_2	...	x_i	x_{n-1}	x_n	x_{n+1}
Probability:	½	¼	...	$1/2^i$	...	$1/2^{n-1}$	$1/2^n$

(9+9)

3.

a) Find the inverse Laplace transform of the following function

$$\frac{1}{s(s+3)^2}.$$

b) An irregular six faced dice is thrown 12 times. The expectation that it will give six even numbers is twice the expectation that it will give 5 even numbers. If 1000 sets, each of exactly 12 trials are made, how many sets are expected not to give any even number?

(9+9)

4.

a) M/s. Mahabir Engineering Works have obtained a large contract for the supply of an alloy steel. The alloy needs three metals, X, Y and Z. The minimum requirement of the metal per week would be 12 units of X, 10 units of Y and 14 units of Z. The metals are available from the dealers who supply them in standardized boxes containing the metals in three different proportions. The boxes are called by code numbers 121, 321 and 115 respectively. Box 121 contains 1 unit of X, 2 units of Y and 1 unit of Z; box 321 contains 3 units of X, 2 units of Y and 1 unit of Z, whereas box 115 contains 1 unit each of X and Y and 5 units of Z. The cost of one box of type 121, 321 and 115, is respectively, Rs.1200, Rs.900 and Rs.1500. Draft its LPP to find no. of boxes of each kind to be bought every week to minimize its cost. Find the dual and give its economic interpretation.

b) A road transport company has one reservation clerk on the time of duty at a time. He handles information of bus schedules and makes reservations. Customer arrive at a rate of 8 per hour and the clerk can service 12 customers on an average per hour. After starting your assumptions, answer the following:

i) What is the average number of customers waiting for the service of the clerk?

ii) What is the average time a customer has to wait before getting the service?

iii) The management is contemplating to install a computer system to handle the information and reservations. This is expected to reduce the service time from 5 to 3 minutes. The additional cost of having the new system works out to Rs. 50 per day. If the cost of goodwill of having to wait, its estimated to be 12 paise per minute spent waiting before being served. Should the company install the computer system? Assume 8 hours working day.

(9+9)

5.

a) Using dynamic programming to solve the following problem:

Minimize $z = y_1^2 + y_2^2 + y_3^2$

subject to the constraints:

$y_1 + y_2 + y_3 \geq 15$ and $y_1, y_2, y_3 \geq 0$.

b) Find the values of x_1, x_2 and x_3 so as to

Maximize $z = -x_1^2 - x_2^2 - x_3^2 + 4x_1 + 6x_2$

subject to the constraints:

$x_1 + x_2 \leq 2$,

$2x_1 + 3x_2 \leq 12$,

$x_1, x_2 \geq 0$

(9+9)

6.

a) An elastic string of length l which is fastened at its ends x = 0 and x = l is released from its horizontal position (zero initial displacement) with initial velocity g(x) given as

$$g(x) = \begin{cases} x, & 0 \leq x \leq l/3 \\ 0, & l/3 \leq x \leq l \end{cases}$$

Find the displacement of the string at any instant of time.

b) The distributive of the number of road accidents per day in a city is Poisson with mean 4. Find the number of days out of 100 days when there will be

i) no accident

ii) atleast 2 accidents

iii) atmost 2 accidents

iv) between 2 and 5 accidents.

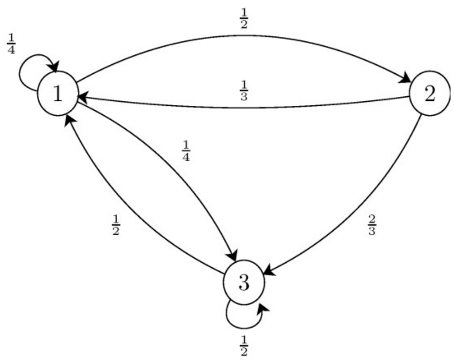
(9+9)

7.

a) Consider the Markov chain shown in the following figure. Assume $X0=1$, and let R be the first time that the chain returns to state 1, i.e.,

$$R=\min\{n\geq 1: X_n=1\}.$$

Find $E[R|X0=1]$.



b) The joint density function of a bivariate distribution is given by

$f(x,y) = 4xy\, e^{-(x^2+y^2)},\ x\geq 0, y \geq 0.$

Find the marginal and conditional probability density functions. Are x and y independent?

(9+9)