«SRLNO»

C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) If the function $f:R\to R$ be given by $f(x)=x^2+2$ and $g:R\to R$ be given by $g(x)=\frac{x}{x-1}$. Find gof and fog.

b) A particle acted by a force $5i+3j+2k$ is displaced from the point $2i-j-3k$ to the point $4i-3j+7k$. Find the work done by the force.

c) For the probability distribution

X	0	1	2
P(*X*)	0.7	0.2	0.1

Find the expected value of *X*.

d) Evaluate $\int\frac{x-1}{(x-2)(x-3)}dx$.

e) Expand $2x^3+7x^2+x-1$ by Taylor's series expansion in powers of $x-2$.

f) For what value of *k* does the line $x+y=1$ touches the parabola $y^2=kx$.

g) A binomial variable *x* satisfies the relation $9P(x=4)=P(x=2)$ when *n* = 6. Find the value of the parameter *P*.

(7x4)

2.

a) Find the Eigen values and Eigen vectors of the matrix $A=\begin{bmatrix}8&-6&2\\-6&7&-4\\2&-4&3\end{bmatrix}$.

b) For what values of *k*, these equation

$$x+y+z=1$$
$$2x+y+4z=k$$
$$4x+y+10z=k^2$$

have a solution and solve them completely.

(9+9)

3.

a) Find $\frac{dy}{dx}$ if $y=(\cos x)^{(\cos x)^{(\cos x)\ldots\ldots\text{to inf}}}$.

b) Find the coefficients of x^{32} and x^{-17} in the expansion of $\left(x^4-\frac{1}{x^3}\right)^{15}$.

c) Find the points of intersection of the line $2x+3y=18$ and the circle $x^2+y^2=25$.

(6+6+6)

4.

a) An urn contains 5 balls. Two balls are drawn at random and are found to be white. What is the probability of all the balls being white?

b) Find the value of *a*, *b* and *c* such that

$$\lim_{x\to 0}\frac{x(a+b\cos x)-c\sin x}{x^5}=1.$$

c) Find the equation of the ellipse whose eccentricity is $\frac{1}{2}$, the focus is $(-1,1)$ and the directrix is $x-y+3=0.$

(6+6+6)

5.

a) If X and Y are two correlated random variables with the same variance and if ν is the correlation coefficient between X and Y, show that the correlation coefficient between X and X+Y is $\sqrt{\frac{1+\nu}{2}}$.

b) Test the convergence of the series

$$\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\frac{1}{4\cdot 5}+\frac{1}{5\cdot 6}+\cdots\cdots\cdots\cdots$$

c) Find the value of *k*, so that the equations $2x^2+kx-5=0$ and $x^2-3x-4=0$ may have one root in common.

(8+6+4)

6.

a) If a random variable X follows Poisson distribution such that $P(x=1)=P(x=2)$ then find (i) the mean of the distribution (ii) $P(X=0)$.

b) A die is thrown 90 times and the number of faces shown is as indicated below:

Faces	1	2	3	4	5	6
Frequency	18	14	13	15	14	16

Test the hypothesis that the die is fair.

(9+9)

7.

a) Solve the following systems of equations by Crammer's Rule

$$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\qquad \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\qquad \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2$$

b) Ten percent of screws produced in a certain factory turn out to be defective. Find the probability that in a sample of 10 screws chosen at random, exactly two will be defective.

c) If $\int_0^k \frac{dx}{2+8x^2}=\frac{\pi}{16}$, find the value of *k*.

(8+6+4)