B4.1-R4: COMPUTER BASED STATISTICAL & NUMERICAL METHODS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**
3. **Only Non-Programmable and Non-Storage type Scientific Calculator allowed.**

Time: 3 Hours **Total Marks: 100**

1.

a) A continuous random variable, X, has the probability density function $f(x)=\begin{cases} kx^2, & 0<x<2 \\ 0, & elsewhere \end{cases}$

Find the value of constant k and $P(X<1)$.

b) Obtain a second degree polynomial approximation to $f(x)=(1+x)^{1/2}$, $x\in[0,\ 0.1]$ using the Taylor series expansion about $x=0$.

c) A box contains 5 red and 4 white balls. Two balls are drawn successively from the box without replacement and it is noted that the second one is white. What is the probability that the first is also white?

d) Find the covariance between X and Y for the following data:

X	10	14	18	22	26	30
Y	18	12	24	6	30	36

e) Evaluate $\int_0^2 e^x dx$ by Simpson's $\frac{1}{3}$ - rule using $h=\frac{2}{5}$ and compare it with the actual values.

f) Find the equation of the cubic curve which passes through the points (4, -43), (7, 83), (9, 327) and (12, 1053).

g) A coin is tossed until a head appears. What is the expected number of tosses?

(7x4)

2.

a) Use Gauss-Seidal iterative method to find the solution of the system of equation, $20x+y-2z=17,\ 3x+20y-z=-18,\ 2x-3y+20z=25.$ {Take initial approximation=(0,0,0)}

b) A survey of 500 families with 4 children has been made regarding the number of boys and girls in the families. The following data was collected

No of families	35	100	200	125	40
No of boys	4	3	2	1	0
No of girls	0	1	2	3	4

Is the data consistent with the hypothesis that the male and female births are equally possible? Test at 5 % level of significance.

(9+9)

3.

a) If a random variable X has the probability density function $f(x)=\begin{cases} 1/4, & |x|\le 2 \\ 0, & otherwise \end{cases}$.

Find i) $P(x<1)$, ii) $P(|x|>1)$, iii) $P(2x+3>5)$.

b) Apply Newton-Raphson method to determine a root of the equation $\cos x - xe^x = 0$ correct to 2 decimal places.

(9+9)

4.

a) Use method of least squares to fit a straight line Y on X and a straight line X on Y for the following data -

x	0	1	2	3
y	2	5	8	11

b) Two weak students attempt to write a program. Their chances of writing the program successfully are 1/8 and 1/12 and the chance of making a common error is 1/1001. Find the chance that the program is correctly written.

(9+9)

5.

a) In a component manufacturing company, there is a probability of 1/500 for any component to be defective. The components are supplied in packets of 10. Use Poisson distribution to calculate the approximate number of packets containing (i) at least one defective, and (ii) at most one defective, in a consignment of 1000 packets.

b) A random variable X has the probability density function $f(x)=\begin{cases}\frac{x+1}{2}, & -1<x<1\\ 0, & otherwise\end{cases}$.

Find mean and variance of X.

(9+9)

6.

a) Solve the following system of equations by the method of Factorization: -

$5x-2y+z=4,$

$7x+y-5z=8,$

$3x+7y+4z=10.$

b) The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75?

(9+9)

7.

a) Find the value of x for which y takes minimum value.

x	3	4	5	6	7	8
y	0.205	0.240	0.259	0.262	0.25	0.224

b) The joint probability density function of random variables X and Y is given by $f(x,y)=\begin{cases}6(1-x-y), & x>0,\ y>0,\ x+y<1\\ 0, & otherwise\end{cases}$.

Find marginal density function of X and Y. Are X and Y independent? Also find $Cov(X,Y)$.

(9+9)