B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Let A, B, and C be any sets. Prove that $(A - C) \cap (C - B) = \emptyset$.

b) Let f and g be functions from set of integers to the set of integers defined by f(x) = 2x + 3 and g(x) = 3x + 2. Find $f \circ g$ and $g \circ f$.

c) If the product of two integers is $2^7 3^8 5^2 7^{11}$ and their greatest common divisor is $2^3 3^4 5$, then what is their least common multiple.

d) If $f(n) = f(n-1) - 1,\ n \geq 1,\ f(0) = 1,$ then determine the formula for $f(n),$ for any non negative integer n.

e) How many edges does a graph have if its degree sequence is 4, 3, 3, 2, 2? Draw such a graph.

f) Find the disjunctive normal form (DNF) of the function $f(x, y, z) = (x + y)\bar{z}$.

g) Let G be a grammar with vocabulary V = { S, 0, 1}, terminals T = {0,1}, starting symbol S, and production P= {S→11S, S→0}. What is the language L(G) of this grammar?

(7x4)

2.

a) Show that $\leftharpoondown (p \to q)\ and\ (p \wedge \leftharpoondown q)$ are logically equivalent.

b) Find an inverse of 19 modulo 141.

c) Draw the graph whose adjacency matrix is given by $\begin{pmatrix} 0 & 2 & 3 & 0 \\ 1 & 2 & 2 & 1 \\ 2 & 1 & 1 & 0 \\ 1 & 0 & 0 & 2 \end{pmatrix}$.

(6+6+6)

3.

a) Solve the recurrence relation together with initial conditions

$$f(n) = 4f(n-1) - 4f(n-2),\ n \geq 2, \qquad f(0) = 6, f(1) = 8.$$

b) Draw the Hasse diagram for divisibility on the set {1, 2, 3, 6, 12, 24, 36, 48, 72}. Also describe the minimal and the maximal element(s) in the lattice.

c) Let m be a positive integer. Prove that the relation $a \equiv b (\bmod m)$ is an equivalence relation on the set of integers.

(8+6+4)

4.

a) Produce a finite state machine that adds two positive integers using their binary expansions.

b) Consider the following random list of 12 numbers say, 2, 4, 6, 8, 11, 15, 23, 34, 55, 67, 78 and 83. Is it possible to choose two of them such that their difference is divisible by 11? Provide an answer by applying the Pigeonhole Principle?

c) Find the least integer n such that f(x) = $2x^3 + x^2 \log(x)$ is of $O(x^n)$.

(8+6+4)

5.

a) Show that $\mathcal{U}(15) = \{ n \mid n\ is\ a\ positive\ integer\ less\ than\ 15\ such\ that\ \gcd(n, 15) = 1\}$, is a group under multiplication modulo 15. Here gcd means the greatest common divisor.

b) The coach of a team offered to buy burgers for players on his team. Of the 44 players in his team, 28 wanted ketchup, 20 wanted mustard sauce, 14 wanted relish, 10 wanted ketchup and mustard, 11 wanted ketchup and relish, 8 wanted mustard and relish and 6 wanted all three condiments. How many players wanted Ketchup and mustard but not relish? How many players want none of the three condiments?

(9+9)

6.

a) Use the Karnaugh map to minimize the sum-of-product expansion
$$x\bar{y}\,z + x\bar{y}\,\bar{z}\; + \bar{x}\,yz + \bar{x}\;\bar{y}\,z + \bar{x}\;\bar{y}\;\bar{z}\,.$$

b) Let W_6 represents a graph called a wheel with 6 vertices in which one vertex is of degree 5 and others are of degree 3 each. Use depth-first search to find the spanning tree of graph W_6, starting from a degree 5 vertex.

(9+9)

7.

a) Prove that if G is a connected planar simple graph then G has a vertex of degree not exceeding five.

b) Use induction to prove that 3 divide $n^3 + 2n$, for every nonnegative integer n.

(9+9)