B0-R4: BASIC MATHEMATICS

NOTE:

1. **Answer question 1 and any FOUR from questions 2 to 7.**
2. **Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Solve the equation $Z^4+1=0$ using De - Moivre's Theorem.

b) Find $\lim_{x\to 0}\left(x^n \log x\right)$.

c) Show that the matrix is orthogonal matrix: $\begin{bmatrix} -2/3 & 1/3 & 2/3 \\ 2/3 & 2/3 & 1/3 \\ 1/3 & -2/3 & 2/3 \end{bmatrix}$.

d) Discuss the convergence of the series $\frac{x}{1+x}+\frac{x^2}{1+x^2}+\frac{x^3}{1+x^3}+\cdots \ (x>0)$.

e) Solve $x^2y\,dx-(x^3+y^3)dy=0$.

f) Determine the length of the cardiode r=a (1 - cosθ) lying outside the circle r = a cosθ.

g) Evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2}dx$.

(7x4)

2.

a) Find the rank of the following matrix:

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 7 \\ 3 & 6 & 10 \end{bmatrix}$$

b) Solve the following system of equations using Cramer's rule:

x- y +z =4
2x +y -3z =0
x+y +z = 2

c) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^2 x dx}{\sin x+\cos x}$.

(4+5+9)

3.

a) Find the asymptotes of the curve $x^3+3x^2y-4y^3-x+y+3=0$.

b) Find the absolute maximum or minimum values of the function

$$f(x)=\sin x(1+\cos x), \quad 0\le x\le 2\pi.$$

(9+9)

4.

a) For the Taylor's polynomial approximation of degree $\le$ n about the point x=0 for the function $f(x)=e^x$, determine the value of n such that the error satisfies $|R_n(x)|\le 0.005$, when $-1\le x\le 1$.

b) Find the area enclosed between the curves $x^2 = 4y$ and the straight line x = 4y – 2.

(9+9)

5.

a) Test the convergence of the series $\sum_1^\infty\left(\sqrt{n^4+1}-\sqrt{n^4-1}\right)$.

b) Let C be a curve defined parametrically as $x = a\cos^3\theta$, $y = a\sin^3\theta$, $0 < \theta < \pi/2$. Determine a point P on C, where the tangent to C is parallel to the chord joining the points (a, 0) and (0, a).

c) Solve $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+5y=e^{-x}$.

(6+6+6)

6.

a) Find the equation of the parabola that is symmetric about the y – axis, has its vertex at the origin, and passes through the point (5, 2).

b) Find the equation for the tangent line to the Folium of Descartes $x^3+y^3 = 3xy$ at the point (3/2, 3/2).

c) Find the angle between the vector u = $\hat{\imath}$ -2$\hat{\jmath}$ + 2$\hat{k}$ and v = -3$\hat{\imath}$ + 6$\hat{\jmath}$ + 2$\hat{k}$.

(6+6+6)

7.

a) Find the parametric equations of the line L passing through the points P_1(2, 4, -1) and P_2(5, 0, 7). Where does the line intersect in the xy - plane?

b) Find the eigen values and eigen vectors of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

and test for the orthogonality of the vectors.

(9+9)