

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) A super market has a single cashier. During peak hours customers arrive at a rate of 20 per hour. The average number of customers that can be serviced by the cashier is 24 per hour. Calculate the probability that the cashier is idle.
- b) There are five jobs, each of which go first over machine 1 and then over machine 2. The processing time (in hours) are given as follows:

Job	Machine 1	Machine 2
A	24	16
B	30	20
C	12	12
D	20	20
E	22	24

Determine the optimum sequence in which job should be processed.

- c) Write the dual of the following linear program:

$$\begin{aligned}
 \max z = & 5x_1 + 6x_2 + 7x_3 \\
 \text{subject to} \quad & x_1 + 2x_2 + x_3 = 10 \\
 & 2x_1 + 3x_2 + 5x_3 \leq 8 \\
 & x_1, x_2, x_3 \geq 0
 \end{aligned}$$

- d) One unit of product A contributes Rs. 7 and requires 3 units of raw materials and 2 hours of labour. One unit of product B contributes Rs. 5 and requires one unit of raw material and 1 hour of labour. Availability of raw material is 48 units and there are 40 hours of labour. Formulate this as a linear program.
- e) Solve the following game involving two players

		B		
		I	II	III
A	I	6	8	6
	II	4	12	2

- f) A manufacturer has to supply his customer 1000 units of product per year. Shortages are not allowed and the storage cost amount to Rs. 1 per unit per year. The set up cost per run is Rs. 100. Find the optimum run size and minimum average yearly cost.
- g) A project consists of 5 activities and the time estimates are given below:

Activity	Optimistic days	Most likely days	Pessimistic days
1-2	4	10	16
1-3	3	6	9
1-4	4	7	16
2-5	5	5	5
3-5	8	11	32

Draw a network diagram and find the critical path and its duration (in days).

(7x4)

2.

- a) Solve the following linear programming problem by revised simplex method

Minimize $x_0 = x_1 + x_2 + x_3$

Subject to

$$x_1 - x_4 - 2x_6 = 5$$

$$x_2 + 2x_2 - 3x_5 + x_6 = 3$$

$$x_3 + 2x_4 - 5x_5 + 6x_6 = 5$$

all $x_i \geq 0$.

- b) Solve the following linear program. Find alternate and all optimal solutions

Maximize $x_0 = 2x_1 + x_2$

Subject to

$$2x_1 - 2x_2 \leq 1$$

$$2x_1 - 3x_2 \leq 1$$

$$2x_1 + x_2 \leq 2$$

$$x_1, x_2 \geq 0$$

(9+9)

3.

- a) There are three operators and four machines. The cost to be paid to operator for running the machine is given in the below table. Solve the cost minimization assignment problem using Hungarian method.

Operator	Machine			
	1	2	3	4
1	7	5	8	4
2	3	6	7	4
3	8	7	9	8

- b) A readymade Garments manufacturer has to process 7 items through two stages of production, i.e., cutting and sewing. The time taken for each of these items at the different stages are given below in appropriate units.

Item -->		1	2	3	4	5	6	7
Processing Time	Cutting	5	7	3	4	6	7	12
	Sewing	2	6	7	5	9	5	8

Find an order in which these items are to be processed through these stages so as to minimize the total processing time.

(9+9)

4.

- a) Formulate the following capital budgeting problem as a zero-one integer programming problem. There are four projects under consideration. Assume that the project run into last three years. Total available funds are 75,000 (to be used at the rate of Rs 25,000/- each year). The expected profit and cost break up is as follows:

Project	Expected profit	Cost		
		Year 1	Year 2	Year 3
1	90,000	8,000	10,000	12,000
2	60,000	2,000	5,000	8,000
3	1,80,000	15,000	10,000	5,000
4	1,00,000	10,000	5,000	5,000

- b) Solve the game whose pay off matrix is given below:

		B			
A	3	-2	4	1	
	-1	4	2	3	
	2	2	6	4	
	2	-4	2	-2	

(9+9)

5.

- a) A steel company has three open furnace and five rolling mills. Transportation cost of shipping steel from furnace to rolling mills are shown in the following table:

	M ₁	M ₂	M ₃	M ₄	M ₅	Capacities
F ₁	4	2	3	2	6	8
F ₂	5	4	5	2	1	12
F ₃	6	5	4	7	3	14
Requirement	4	4	6	8	8	

What is the optimal shipping strategy?

- b) A purchase manager has decided to place order to a minimum quantity of 500 numbers of a particular item in order to get a discount of 10%. From the past records, it was found that in the last year, 8 orders each of size 200 units were placed. The ordering cost is 40% of the inventory value and the price of the item is Rs. 400/- per unit. Is the purchase manager justified in his decision? What is the effect of his decision on saving to the company?

(9+9)

6.

- a) Two repairmen are attending five machines in a workshop. Each machine breaks down according to Poisson distribution with mean 3 per hour. The repair time per machine is exponential with mean 15 minutes.

- Find the probability that two repairmen are idle.
- What is the expected number of idle machines not being served?

- b) A project consist of a series of tasks labeled A, B, C, ..., H, I, with the following relationships (w < x,y implies that x and y can not start until w is completed). With this notation construct the network diagram having the following relations:

A < D, E; B,D < F; C < G; B < H; F,G < I.

Find also the minimum time of completion of the project, when the time in days of completion of each task is as follows:

Task:	A	B	C	D	E	F	G	H	I
Time:	23	8	20	16	24	18	19	4	10

(8+10)

7.

- a) A contractor of second hand motor trucks uses to maintain a stock of trucks every month. The demand of trucks occurs at a relatively constant rate but not in a constant size. The demand follows the following probability distribution

Demand (r) :	0	1	2	3	4	5	6 or more
Probability (p(r)):	.40	.24	.20	.10	.05	.01	.00

The holding cost of an old truck for one month is Rs 100.00 and penalty for a truck if not supplied on the demand is Rs 1000.00. Determine optimal size of the stock for the contractor.

- b) Consider $\text{Min } f(X) = x_1^2 - x_1 x_2 + 3 x_2^2$, given initial point $X_1 = (1,2)^T$. Perform two complete iterations of the steepest descent method starting from X_1 .

(9+9)