B3.2-R4: DISCRETE STRUCTURE

NOTE:

1.	**Answer question 1 and any FOUR from questions 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) Prove that for any sets A,B,C, if $A \cap C = B \cap C$ and $A \cup C = B \cup C$, then $A = B$.

b) Show that f: $[2,7] \to R$ defined as $f(x) = 4x^2 - 5x - 9$ is a one-one function.

c) Let A = {1,2,3,4,5,6} and R be a relation on A defined by a R b if and only if a is a multiple of b. Represent the relation R as a matrix and draw its diagram.

d) Find the number of edges of a 4-regular graph with 6 vertices.

e) Given the permutation p:

P = $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 3 & 1 & 5 & 6 \end{pmatrix}$ compute p^{-1}.

f) A collection of 10 electric bulbs contains 3 defective ones. In how many ways can a sample of 4 bulbs be selected which contain 2 good and 2 defective bulbs?

g) Without using truth table prove that $(p \to q) \wedge (q \to r) \to (p \to r)$ is a tautology.

(7x4)

2.

a) Solve the following recurrence relation $x_n - 3x_{n-1} + 2x_{n-2} = 2^n$.

b) Prove that $\forall x\,(P(x) \to Q(x)) \Rightarrow \forall x\, P(x) \to \forall x\, Q(x)$.

c) Is $2^{2n} = O(2^n)$? Give reasons for your answer.

(6+6+6)

3.

a) How many integral solution are possible for system of equations

$$x_1 + x_2 + x_3 + x_4 + x_5 = 20$$
$$x_1 + x_2 = 15,$$

each $x_i \geq 0$, $i = 1, 2, 3, 4, 5$

b) 'A' can hit a target four times in five shots; 'B' can hit the same target three times in four shots; and 'C' can hit the target two times in three shots. Each of them fire one shot at a target. What is the probability that at least two shots hit the target?

c) If X is a number of tails in three tosses of a fair coin then find the expedition E(X) of X.

(7+7+4)

4.

a) If L1 and L2 are regular languages over Σ. Show that $L1 \cap L2$ is regular.

b) Write a regular expression for $L = \{w \in \{a,b\}^*: n_b(w) \equiv 2 \bmod 3\}$ and construct a finite automata that accept it.

c) Define pushdown automata. Construct a NPDA that accepts the language $\{w \in \{a,b\}^+, n_a(w) = n_b(w)\}$

(6+6+6)

5.

a) Simplify the Boolean expression using the Karnaugh map.

$$E(x_1,x_2,x_3) = x_1' x_2' x_3' + x_1' x_2 x_3' + x_1 x_2 x_3' + x_1 x_2' x_3$$

b) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$; use mathematical induction to show that, for all $n \in N$,

$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$.

c) Use pigeonhole principle, show that if any five integers are chosen from 1 to 8, then two of them always add to 9.

(6+6+6)

6.

a) If H is a subgroup of G such that $x^2 \in H$ for every $x \in G$, then prove that H is a normal subgroup of G.

b) Write Lagrange's theorem and use it to show that any group of prime order can have no proper subgroup.

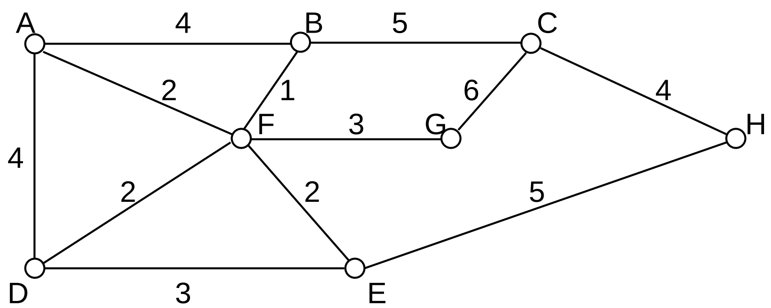
c) Let S={1, 2, 3, …, 11, 12} and define a relation R on it as xRy if and only if y is a multiple of x, where $x,y \in S$. Draw the Hassee diagram of (S, R). Is (S, R) a lattice? Justify.

(6+6+6)

7.

a) Give an example of a graph that has
   i) neither an Euler circuit nor a Hamiltonian cycle
   ii) An Euler circuit but no Hamiltonian cycle
   iii) A Hamiltonian cycle but not an Euler circuit.

b) Find a minimum spanning tree of a weighted graph below using the Kruskal's algorithm. Explain the steps



c) A tree has two vertices of degree 2, one vertex of degree 3 and three vertices of degree 4.How many vertices of degree 1 does it have if it has no vertex of degree greater than 4?

(6+6+6)