

C0-R4.B1: ELEMENTS OF MATHEMATICAL SCIENCES

NOTE:

1. Answer question 1 and any FOUR from questions 2 to 7.
2. Parts of the same question should be answered together and in the same sequence.

Time: 3 Hours

Total Marks: 100

1.

- a) Find the probability of getting 4 heads in 6 tosses of a fair coin.
- b) If $\vec{a} = 5i - 4j + k$ and $\vec{b} = -4i + 3j - 2k$ then find $\vec{a} \cdot \vec{b}$ and $\vec{a} \times \vec{b}$.
- c) Identify the following statement true or false, Justify your answer.
"A binomial distribution can have mean 5 and standard deviation 3".
- d) Evaluate the integral, $\int \frac{1}{\sqrt{x}} \cos \sqrt{x} dx$.
- e) Find the first three terms of the Maclaurin's series for $f(x) = \tan^{-1} x$.
- f) Find the equation of the parabola whose focus is (1,-1) and vertex is (2,1).
- g) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^{2n}}{n!}$.

(7x4)

2.

- a) Solve the following system of equations by Crammer's Rule:

$$x + 2y + 3z = 6$$

$$2x + 4y + z = 7$$

$$3x + 2y + 9z = 14$$

- b) Find the inverse of the matrix $\begin{bmatrix} 0 & 0 & -1 \\ 3 & 4 & 5 \\ -2 & -4 & -7 \end{bmatrix}$ by Gauss Elimination method.

(9+9)

3.

- a) If $x = a(\cos \theta + \theta \sin \theta)$, $y = a(\sin \theta - \theta \cos \theta)$, then find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.
- b) Discuss the applicability of the Lagrange mean value theorem for the function $f(x) = x(x-1)(x-2)$ in the interval $\left[0, \frac{1}{2}\right]$.
- c) Find the limit when $n \rightarrow \infty$ of the series $\frac{1}{n} + \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{3n}$.

(6+6+6)

4.

- a) Find the equation of a circle which passes through the points $(1, -2)$ and $(4, -3)$ and has its centre on the line $3x + 4y = 7$.

b) Evaluate $\int_0^{\pi/4} \frac{\sec x}{1 + 2\sin^2 x} dx$.

- c) Find all the asymptotes of the curve $y^2(x - 2) = x^2(y - 1)$.

(6+6+6)

5.

- a) In a bolt factory, machines A, B and C manufacture respectively 25%, 35% and 40% of the total production. Let from their output, 5%, 4% and 2%, are defective bolts respectively. A bolt is drawn at random from the product and is found to be defective. What is the probability that it was manufactured by machine B?

- b) Test the convergence of an infinite series whose n^{th} term is $\frac{\sqrt{n}}{n^2 + 1}$.

c) Evaluate $\lim_{x \rightarrow 0} \frac{\log x}{\cot x}$.

(8+6+4)

6.

- a) Find the latus rectum, the eccentricity, coordinates of foci and length of the axes of the ellipse $3x^2 + 4y^2 = 12$.

b) Find the Eigen values and Eigen vectors of $\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$.

(9+9)

7.

- a) Let the probability density function of a random variable x be

$$f(x) = \begin{cases} kx^{\alpha-1}(1-x)^{\beta-1}, & \text{for } 0 < x < 1, \alpha > 0, \beta > 0 \\ 0, & \text{otherwise} \end{cases}$$

Find k and mean of x .

- b) Find the regression line y on x for the following data:

x	1	3	4	6	8	9	11	14
y	1	2	4	4	5	7	8	9

Also, estimate the value of y , when $x = 10$.

(9+9)